Name ______________________ Class ____________ Date ____________

5-1 Additional Vocabulary Support

Polynomial Functions

Match each word in Column A with the matching polynomial in Column B.

Column A	Column B
1. cubic	**A.** 8
2. linear	**B.** $3x^4 + 5x^2 - 1$
3. quartic	**C.** $2x^2 - 2$
4. quintic	**D.** $7x^3 + 3x^2 + 4$
5. constant	**E.** $x + 10$
6. quadratic	**F.** $6x^5 + 3x^3 + 11x + 3$

Match each polynomial in Column A with the matching word in Column B.

Column A	Column B
7. $5x^3 + 7x$ C	**A.** trinomial
8. $4x^5 + 6x^2 + 3$ A	**B.** monomial
9. $8x^4$ B	**C.** binomial

Use the words from the lists below to name each polynomial by its degree and its number of terms.

Degree				
linear	quadratic	cubic	quartic	quintic

Number of Terms		
monomial	binomial	trinomial

10. $4x^2 - 2x + 3$ quadratic, trinomial

11. $6x^3$ cubic, monomial

12. $3x^5 + 7x^3 - 4$ quintic, trinomial

13. $8x + 3$ linear, binomial

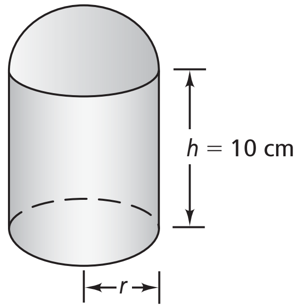
14. $2x^4 + 5x^2$ quartic, binomial

5-1 Think About a Plan

Polynomial Functions

Packaging Design The diagram at the right shows a cologne bottle that consists of a cylindrical base and a hemispherical top.

a. Write an expression for the cylinder's volume.
b. Write an expression for the volume of the hemispherical top.
c. Write a polynomial to represent the total volume.



1. What is the formula for the volume of a cylinder? Define any variables you use in your formula.

$V_c = \boxed{\pi r^2 h}$, where r is the radius of the base and h is the height.

2. Write an expression for the volume of the cylinder using the information in the diagram.

$V_c = \boxed{10\pi r^2}$

3. What is the formula for the volume of a sphere? Define any variables you use in your formula.

$V_s = \boxed{\frac{4}{3}\pi r^3}$, where r is the radius of the sphere.

4. Write an expression for the volume of the hemisphere.

$V_h = \boxed{\frac{2}{3}\pi r^3}$

5. How can you find the total volume of the bottle?

Add the volume of the cylinder and the volume of the hemisphere.

6. Write a polynomial expression to represent the total volume of the bottle.

$V = \boxed{10\pi r^2} + \boxed{\frac{2}{3}\pi r^3}$

7. Is the polynomial expression you wrote in simplest form? Explain.

Yes; you cannot combine the terms because they are not like terms.

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5-1 Practice

Form G

Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. $4x + x + 2$
   $5x + 2$; linear binomial
2. $-3 + 3x - 3x$
   -3; constant monomial
3. $6x^4 - 1$
   $6x^4 - 1$; quartic binomial
4. $1 - 2s + 5s^4$
   $5s^4 - 2s + 1$; quartic trinomial
5. $5m^2 - 3m^2$
   $2m^2$; quadratic monomial
6. $x^2 + 3x - 4x^3$
   $-4x^3 + x^2 + 3x$; cubic trinomial
7. $-1 + 2x^2$
   $2x^2 - 1$; quadratic binomial
8. $5m^2 - 3m^3$
   $-3m^3 + 5m^2$; cubic binomial
9. $5x - 7x^2$
   $-7x^2 + 5x$; quadratic binomial
10. $2 + 3x^3 - 2$
    $3x^3$; cubic monomial
11. $6 - 2x^3 - 4 + x^3$
    $-x^3 + 2$; cubic binomial
12. $6x - 7x$
    $-x$; linear monomial
13. $a^3(a^2 + a + 1)$
    $a^5 + a^4 + a^3$; quintic trinomial
14. $x(x + 5) - 5(x + 5)$
    $x^2 - 25$; quadratic binomial
15. $p(p - 5) + 6$
    $p^2 - 5p + 6$; quadratic trinomial
16. $(3c^2)^2$
    $9c^4$; quartic monomial
17. $-(3 - b)$
    $b - 3$; linear binomial
18. $6(2x - 1)$
    $12x - 6$; linear binomial
19. $\frac{2}{3} + s^2$
    $s^2 + \frac{2}{3}$; quadratic binomial
20. $\frac{2x^4 + 4x - 5}{4}$
    $\frac{1}{2}x^4 + x - \frac{5}{4}$; quartic trinomial
21. $\frac{3 - z^5}{3}$
    $-\frac{1}{3}z^5 + 1$; quintic binomial

Determine the end behavior of the graph of each polynomial function.

22. $y = 3x^4 + 6x^3 - x^2 + 12$
    up and up
23. $y = 50 - 3x^3 + 5x^2$
    up and down
24. $y = -x + x^2 + 2$
    up and up
25. $y = 4x^2 + 9 - 5x^4 - x^3$
    down and down
26. $y = 12x^4 - x + 3x^7 - 1$
    down and up
27. $y = 2x^5 + x^2 - 4$
    down and up
28. $y = 5 + 2x + 7x^2 - 5x^3$
    up and down
29. $y = 20 - 5x^6 + 3x - 11x^3$
    down and down
30. $y = 6x + 25 + 4x^4 - x^2$
    up and up

Describe the shape of the graph of each cubic function by determining the end behavior and number of turning points.

31. $y = x^3 + 4x$
    down and up; no turns
32. $y = -2x^3 + 3x - 1$
    up and down; two turns
33. $y = 5x^3 + 6x^2$
    down and up; two turns

Determine the degree of the polynomial function with the given data.

34.

x	y
−2	−16
−1	1
0	4
1	5
2	16

3rd degree

35.

x	y
−2	52
−1	6
0	2
1	4
2	48

4th degree

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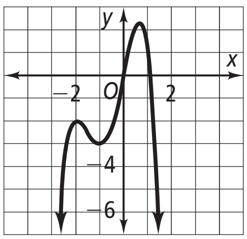
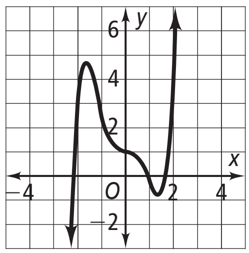
5-1 Practice (continued) — Form G

Polynomial Functions

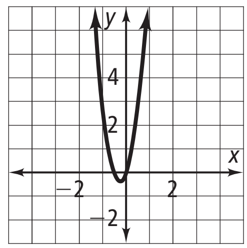
Determine the sign of the leading coefficient and the degree of the polynomial function for each graph.

36. 

negative; 4th degree

37. 

positive; 5th degree

38. 

positive; 2nd degree

39. Error Analysis A student claims the function $y = 3x^4 - x^3 + 7$ is a fourth-degree polynomial with end behavior of down and down. Describe the error the student made. What is wrong with this statement? **The degree is even and the leading coefficient is positive, so the end behavior should be up and up.**

40. The table at the right shows data representing a polynomial function.

a. What is the degree of the polynomial function? **5th degree**

b. What are the second differences of the y-values? **−726, −126, −6, 114, 714**

c. What are the differences when they are constant? **480**

x	y
−3	−999
−2	−140
−1	−7
0	0
1	1
2	116
3	945

Classify each polynomial by degree and by number of terms. Simplify first if necessary.

41. $4x^5 - 5x^2 + 3 - 2x^2$
5th degree; 3 terms

42. $b(b - 3)^2$
3rd degree; 3 terms

43. $(7x^2 + 9x - 5) + (9x^2 - 9x)$
2nd degree; 2 terms

44. $(x + 2)^3$
3rd degree; 4 terms

45. $(4s^4 - s^2 - 3) - (3s - s^2 - 5)$
4th degree; 3 terms

46. 13
0 degree; 1 term

47. Open-Ended Write a third-degree polynomial function. Make a table of values and a graph. **Check students' work.**

48. Writing Explain why finding the degree of a polynomial is easier when the polynomial is written in standard form. **When a polynomial is written in standard form, the term with the greatest exponent becomes the first term. This exponent is equal to the degree of the polynomial.**

Name ______________________ Class ____________ Date ____________

5-1 Practice

Form K

Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. $4x^3 - 3 + 2x^2$

To start, write the terms of the polynomial with their degrees in descending order. $4x^3 + 2x^2 - 3$

3rd degree, 3 terms

2. $8 - x^5 + 9x^2 - 2x$

$-x^5 + 9x^2 - 2x + 8$; **5th degree, 4 terms**

3. $6x + 2x^4 - 2$

$2x^4 + 6x - 2$; **4th degree, 3 terms**

4. $-6x^3$

3rd degree, 1 term

5. $3 + 24x^2$

$24x^2 + 3$; **2nd degree, 2 terms**

Determine the end behavior of the graph of each polynomial function.

6. $y = 5x^3 - 2x^2 + 1$

down and up

7. $y = 5 - x + 4x^2$

up and up

8. $y = x - x^2 + 10$

down and down

9. $y = 3x^2 + 9 - x^3$

up and down

10. $y = 8x^2 - 4x^4 + 5x^7 - 2$

down and up

11. $y = 20 - x^5$

up and down

12. $y = 1 + 2x + 4x^3 - 8x^4$

down and down

13. $y = 15 - 5x^6 + 2x - 22x^3$

down and down

14. $y = 3x + 10 + 8x^4 - x^2$

up and up

Describe the shape of the graph of each cubic function by determining the end behavior and number of turning points.

15. $y = x^3 + 2x$

To start, make a table of values to help you sketch the middle part of the graph.

x	y
−2	−12
−1	−3
0	0
1	3
2	12

down and up, 0 turning points

16. $y = -3x^3 + 4x^2 - 1$

up and down, 2 turning points

17. $y = 4x^3 + 2x^2 - x$

down and up, 2 turning points

Determine the degree of the polynomial function with the given data.

18.

x	y
−3	−43
−2	−10
−1	1
0	2
1	5
2	22
3	65

3rd degree

19.

x	y
−3	65
−2	5
−1	−5
0	−1
1	5
2	25
3	95

4th degree

Name ______________________ Class ____________ Date ____________

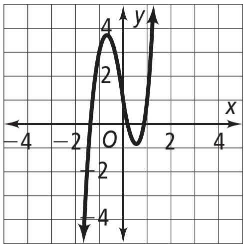
5-1 Practice (continued)

Form K

Polynomial Functions

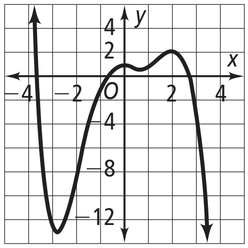
Determine the sign of the leading coefficient and the degree of the polynomial function for each graph.

20.



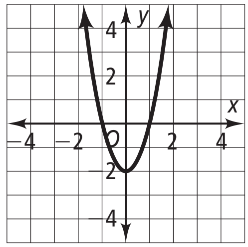
positive, 3rd degree

21.



negative, 5th degree

22.



positive, 2nd degree

23. Error Analysis A student claims the function $y = -2x^3 + 5x - 7$ is a 3rd degree polynomial with ending behavior of down and up. Describe the error the student made. What is wrong with this statement?
The degree is odd and the leading coefficient is negative, so the ending behavior should be up and down.

24. The table to the right shows data representing a polynomial function.

x	y
−3	98
−2	20
−1	6
0	2
1	2
2	48
3	230

a. What is the degree of the polynomial function? 4th degree

b. What are the second differences of the y-values? 64, 10, 4, 46, 136

c. What are the differences when they are constant? 48

Classify each polynomial by degree and by number of terms. Simplify first if necessary.

25. $3x^5 - 6x^2 - 5 + x^2$
5th degree, 3 terms

26. $a - 2a + 3a^2$
2nd degree, 2 terms

27. $(5x^2 + 2x - 8) + (5x^2 - 4x)$
2nd degree, 3 terms

28. $c^3(5 - c^2)$
5th degree, 2 terms

29. $(5s^3 - 2s^2) - (s^4 + 1)$
4th degree, 4 terms

30. $x(3x)(x + 2)$
3rd degree, 2 terms

31. $(2s - 1)(3s + 3)$
2nd degree, 3 terms

32. 5
0 degree, 1 term

33. Open-Ended Write a fourth-degree polynomial function. Make a table of values and a graph. Check students' work.

5-1 Standardized Test Prep

Polynomial Functions

Multiple Choice

For Exercises 1–7, choose the correct letter.

1. Which expression is a binomial? D

(A) $2x$ (B) $\frac{x}{2}$ (C) $3x^2 + 2x + 4$ (D) $x - 9$

2. Which polynomial function has an end behavior of up and down? F

(F) $-6x^7 + 4x^2 - 3$ (H) $6x^7 - 4x^2 + 3$

(G) $-7x^6 + 3x - 2$ (I) $7x^6 - 3x + 2$

3. What is the degree of the polynomial $5x + 4x^2 + 3x^3 - 5x$? C

(A) 1 (B) 2 (C) 3 (D) 4

4. What is the degree of the polynomial represented by the data in the table at the right? G

(F) 2 (G) 3 (H) 4 (I) 5

x	y
−3	77
−2	24
−1	1
0	−4
1	−3
2	−8
3	−31

5. For the table of values at the right, if the nth differences are constant, what is the constant value? B

(A) −12 (B) 12 (C) 1 (D) 6

6. What is the standard form of the polynomial $9x^2 + 5x + 27 + 2x^3$? I

(F) $27 + 5x + 9x^2 + 2x^3$ (H) $9x^2 + 5x + 27 + 2x^3$

(G) $9x^2 + 5x + 2x^3 + 27$ (I) $2x^3 + 9x^2 + 5x + 27$

7. What is the number of terms in the polynomial $(2a - 5)(a^2 - 1)$? C

(A) 2 (B) 3 (C) 4 (D) 5

Short Response

8. Simplify $(9x^3 - 4x + 2) - (x^3 + 3x^2 + 1)$. Then name the polynomial by degree and the number of terms.

[2] $8x^3 - 3x^2 - 4x + 1$; 3rd degree; 4 terms
[1] simplified polynomial is correct, but degree and/or number of terms is incorrect OR simplified polynomial is incorrect, but degree and terms are correct for the polynomial given.
[0] no answers given

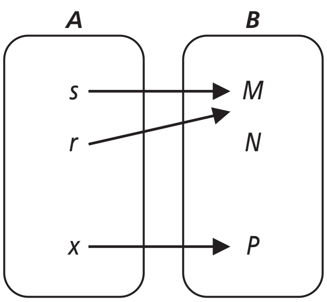
Name ______________________ Class ____________ Date ____________

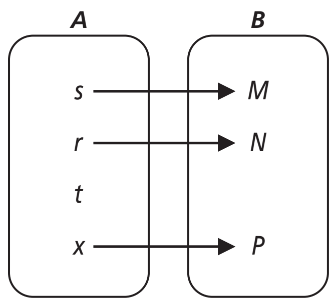
5-1 Enrichment

Polynomial Functions

Mathematicians use precise language to describe the relationships between sets. One important relationship is described as a function. You have graphed polynomial functions. Using this one word may not seem important, but it describes a very specific relationship between the domain and range of a polynomial. The word function tells you that every element of the domain corresponds with exactly one element of the range.

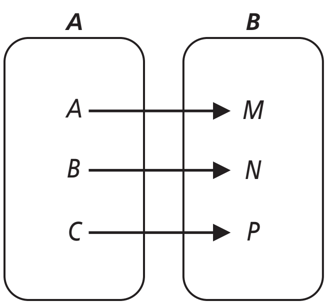
1. Another important relationship between two sets is described by the word *onto*. A function from set A to set B is onto if every element in set B is matched with an element in set A. Which of the following relations shows a function from set A to set B that is onto? Explain.

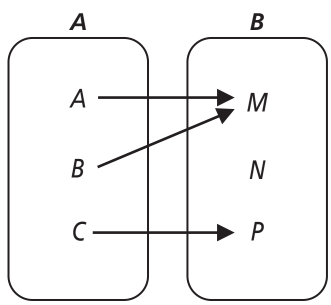




The second relation is a function that is onto because all of the elements in set *B* are matched with an element in set *A*.

2. Another relationship between two sets is described as *one-to-one*. A function from set A to set B is one-to-one if no element of set B is paired with more than one element of set A. Which of the following relations shows a function from set A to set B that is one-to-one? Explain.





The first relation is a one-to-one function because all of the elements of set *B* are paired with exactly one element of set *A*.

Describe each polynomial function. If it is not possible, explain why.

3. Describe a polynomial function that is onto but not one-to-one. **Answers may vary. Sample: A cubic with two turning points is onto because every *y*-value in the range is paired with an *x*-value, but not one-to-one because some *y*-values are paired with more than one *x*-value.**

4. Is there a polynomial function that is one-to-one but not onto? **no; Answers may vary. Sample: Odd degree polynomials are one-to-one, but they are also onto.**

5. Describe a polynomial function that is both onto and one-to-one. **Answers may vary. Sample: The cubic $y = x^3$ is onto because every *y*-value is paired with an *x*-value, and it is one-to-one because every *y*-value is paired with exactly one *x*-value.**

Name ______________________ Class ______________ Date ______________

5-1 Reteaching

Polynomial Functions

Problem

What is the classification of the following polynomial by its degree? by its number of terms? What is its end behavior? $5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

Step 1 Write the polynomial in standard form. First, combine any like terms. Then, place the terms of the polynomial in descending order from greatest exponent value to least exponent value.

$5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

$8x^4 - 3x + 3x^6 + 9x^3 - 12$ Combine like terms.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ Place terms in descending order.

Step 2 The degree of the polynomial is equal to the value of the greatest exponent. This will be the exponent of the first term when the polynomial is written in standard form.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ The first term is $3x^6$.

$3x^6$ The exponent of the first term is 6.

This is a sixth-degree polynomial.

Step 3 Count the number of terms in the simplified polynomial. It has 5 terms.

Step 4 To determine the end behavior of the polynomial (the directions of the graph to the far left and to the far right), look at the degree of the polynomial (n) and the coefficient of the leading term (a).

If a is positive and n is even, the end behavior is up and up.

If a is positive and n is odd, the end behavior is down and up.

If a is negative and n is even, the end behavior is down and down.

If a is negative and n is odd, the end behavior is up and down.

The leading term in this polynomial is $3x^6$.
a $(+3)$ is positive and n (6) is even, so the end behavior is up and up.

Exercises

What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?

1. $8 - 6x^3 + 3x + x^3 - 2$
3rd degree; 3 terms; up and down

2. $15x^7 - 7$
7th degree; 2 terms; down and up

3. $2x - 6x^2 - 9$
2nd degree; 3 terms; down and down

Name ______________________ Class ____________ Date ____________

5-1 Reteaching (continued)

Polynomial Functions

Problem

x	y
−3	52 (y_1)
−2	18 (y_2)
−1	2 (y_3)
0	−2 (y_4)
1	0 (y_5)
2	2 (y_6)
3	−2 (y_7)

What is the degree of the polynomial function that generates the data shown at the right? What are the differences when they are constant?

To find the degree of a polynomial function from a data table, you can use the differences of the y-values.

Step 1 Determine the values of $y_2 - y_1, y_3 - y_2, y_4 - y_3, y_5 - y_4, y_6 - y_5, y_7 - y_6$. These are called the first differences. Make a new column using these values.

Step 2 Continue determining differences until the y-values are all equal. The quantity of differences is the degree of the polynomial function.

The third differences are all equal so this is a third degree polynomial function. The value of the third differences is −6.

x	y	1st diff.
−3	52 (y_1)	−34 (y_8)
−2	18 (y_2)	−16 (y_9)
−1	2 (y_3)	−4 (y_{10})
0	−2 (y_4)	2 (y_{11})
1	0 (y_5)	2 (y_{12})
2	2 (y_6)	−4 (y_{13})
3	−2 (y_7)	

x	y	1st diff.	2nd diff.	3rd diff.
−3	52 (y_1)	−34 (y_8)	18	−6
−2	18 (y_2)	−16 (y_9)	12	−6
−1	2 (y_3)	−4 (y_{10})	6	−6
0	−2 (y_4)	2 (y_{11})	0	−6
1	0 (y_5)	2 (y_{12})	−6	
2	2 (y_6)	−4 (y_{13})		
3	−2 (y_7)			

Exercises

What is the degree of the polynomial function that generates the data in the table? What are the differences when they are constant?

4.

x	y
−3	216
−2	24
−1	0
0	0
1	0
2	−24
3	−216

5th degree; −120

5.

x	y
−3	−101
−2	−37
−1	−11
0	−5
1	−1
2	19
3	73

3rd degree; 18

6.

x	y
−3	6
−2	26
−1	8
0	0
1	2
2	−34
3	−204

4th degree; −48

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5-2 Additional Vocabulary Support

Polynomials, Linear Factors, and Zeros

There are two sets of note cards below that show how to write a cubic polynomial function with zeros −3, 1, and 4 in standard form. The set on the left explains the thinking. The set on the right shows the steps. Write the thinking and the steps in the correct order.

Think Cards

Multiply $(x - 1)$ and $(x - 4)$.

Use the Distributive Property to multiply $(x + 3)$ and $(x^2 - 5x + 4)$.

Simplify.

Write a linear factor for each zero.

Write Cards

$x(x^2 - 5x + 4) + 3(x^2 - 5x + 4)$
$= x^3 - 5x^2 + 4x + 3x^2 - 15x + 12$

$(x + 3)(x^2 - 5x + 4)$

$x^3 - 2x^2 - 11x + 12$

$f(x) = (x + 3)(x - 1)(x - 4)$

Think	Write
First, you should write a linear factor for each zero.	**Step 1** $f(x) = (x + 3)(x - 1)(x - 4)$
Second, you should multiply $(x - 1)$ and $(x - 4)$.	**Step 2** $(x + 3)(x^2 - 5x + 4)$
Then, you should use the Distributive Property to multiply $(x + 3)$ and $(x^2 - 5x + 4)$.	**Step 3** $x(x^2 - 5x + 4) + 3(x^2 - 5x + 4) = x^3 - 5x^2 + 4x + 3x^2 - 15x + 12$
Finally, you should simplify.	**Step 4** $x^3 - 2x^2 - 11x + 12$

Name ______________________ Class __________ Date __________

5-2 Think About a Plan

Polynomials, Linear Factors, and Zeros

Measurement The volume in cubic feet of a CD holder can be expressed as $V(x) = -x^3 - x^2 + 6x$, or, when factored, as the product of its three dimensions. The depth is expressed as $2 - x$. Assume that the height is greater than the width.

a. Factor the polynomial to find linear expressions for the height and the width.
b. Graph the function. Find the x-intercepts. What do they represent?
c. What is a realistic domain for the function?
d. What is the maximum volume of the CD holder?

1. What do you know about the factors of the polynomial?

One of the factors is $(2-x)$.

2. Factor the polynomial.

$V(x) = -x^3 - x^2 + 6x = \left(\boxed{2 - x}\right)\left(\boxed{x}\right)\left(\boxed{x + 3}\right)$

3. What are the height and width of the CD holder? How do you know which factor is the height and which factor is the width?

height: $x + 3$; width: x; the height is greater than the width, and $x + 3$ is greater than x.

4. Graph the function on a graphing calculator. How can you find the x-intercepts?

Set the factors equal to zero and solve or use the zero feature.

5. What are the x-intercepts? What do they represent?

$-3, 0, 2$; values of x that result in a volume of 0.

6. What are the limits of each of the factors? What is a realistic domain for the function? Explain.

Each dimension of the CD holder must be greater than 0, so each factor should be greater than 0. So $2 - x > 0$, $x > 0$, and $x + 3 > 0$. The solution of all three inequalities is $0 < x < 2$.

7. How can you find the maximum volume of the CD holder?

Find the maximum value of the function in the domain $0 < x < 2$. Use the maximum feature of the CALC menu with LeftBound = 0 and RightBound = 2.

8. What is the maximum volume of the CD holder? about 4.06 ft^3

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5-2 Practice

Form G

Polynomials, Linear Factors, and Zeros

Write each polynomial in factored form. Check by multiplication.

1. $2x^3 + 10x^2 + 12x$ **$2x(x + 2)(x + 3)$**

2. $x^4 - x^3 - 6x^2$ **$x^2(x + 2)(x - 3)$**

3. $-3x^3 + 18x^2 - 27x$ **$-3x(x - 3)^2$**

4. $x^3 - 2x^2 + x$ **$x(x - 1)^2$**

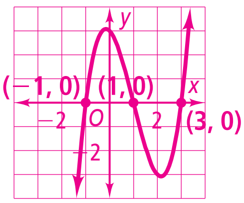
5. $x^3 + 7x^2 + 15x + 9$ **$(x + 3)^2(x + 1)$**

6. $2x^4 + 23x^3 + 60x^2 - 125x - 500$ **$(x + 4)(2x - 5)(x + 5)^2$**

Find the zeros of each function. Then graph the function.

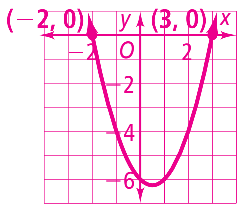
7. $y = (x + 1)(x - 1)(x - 3)$

$-1, 1, 3$



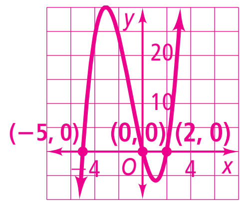
8. $y = (x + 2)(x - 3)$

$-2, 3$



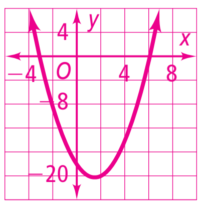
9. $y = x(x - 2)(x + 5)$

$-5, 0, 2$



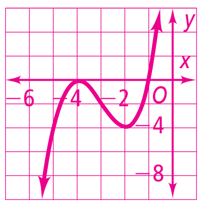
10. $y = (x - 6)(x + 3)$

$6, -3$



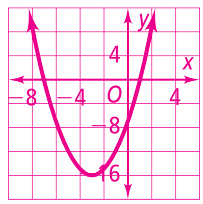
11. $y = (x + 4)^2(x + 1)$

$-4, -4, -1$



12. $y = (x - 1)(x + 7)$

$1, -7$



Write a polynomial function in standard form with the given zeros.

13. $x = -1, 3, 4$
$y = x^3 - 6x^2 + 5x + 12$

14. $x = 1, 1, 2$
$y = x^3 - 4x^2 + 5x - 2$

15. $x = -3, 0, 0, 5$
$y = x^4 - 2x^3 - 15x^2$

16. $x = 4, 2, -3, 0$
$y = x^4 - 3x^3 - 10x^2 + 24x$

17. $x = -1, 5, -2$
$y = x^3 - 2x^2 - 13x - 10$

18. $x = -6, 0$
$y = x^2 + 6x$

Find the zeros of each function. State the multiplicity of multiple zeros.

19. $y = (x - 5)^3$
5, multiplicity 3

20. $y = x(x - 8)^2$
0; 8, multiplicity 2

21. $y = (x - 2)(x + 7)^3$
2; −7, multiplicity 3

22. $y = x^4 - 8x^3 + 16x^2$
0, multiplicity 2; 4, multiplicity 2

23. $y = 9x^3 - 81x$
−3, 0, 3

24. $y = (2x + 5)(x - 3)^2$
$-\frac{5}{2}$; 3, multiplicity 2

5-2 Practice (continued) *Form G*

Polynomials, Linear Factors, and Zeros

Find the relative maximum and relative minimum of the graph of each function.

25. $f(x) = x^3 - 7x^2 + 10x$
rel. max.: 4.06; rel. min.: −8.21

26. $f(x) = x^3 - x^2 - 9x + 9$
rel. max.: 16.9; rel. min.: −5.05

27. $f(x) = x^4 + x^3 - 3x^2 - 5x - 2$
rel. min.: about −8.54

28. $f(x) = x^2 - 6x + 9$
rel. min.: 0

29. A rectangular box has a square base. The combined length of a side of the square base, and the height is 20 in. Let x be the length of a side of the base of the box.

a. Write a polynomial function in factored form modeling the volume V of the box. $\boldsymbol{V = x^2(20 - x)}$

b. What is the maximum possible volume of the box? **about 1185 in.^{3}**

30. Reasoning A polynomial function has a zero at $x = -2a$. Find one of its factors. $\boldsymbol{(x + 2a)}$

31. The side of a cube measures $3x + 2$ units long. Express the volume of the cube as a polynomial. $\boldsymbol{27x^3 + 54x^2 + 36x + 8}$

32. Writing The volume of a box is $x^3 - 3x^2 + 3x - 1$ cubic units. Explain how to find the length of a side if the box is a cube. **The factors of this polynomial are $(x - 1)(x - 1)(x - 1)$. Because all three factors are equal, and the sides of the box are equal, the side length would be $x - 1$.**

33. You have a block of wood that you want to use to make a sculpture. The block is currently $3x$ units wide, $4x$ units high, and $5x$ units deep. You need to remove 1 unit from each dimension before you can begin your sculpture.

a. What is the original volume of the block? **$60x^3$ cubic units**

b. What is the new volume of the block? **$60x^3 - 47x^2 + 12x - 1$ cubic units**

c. What is the volume of the wood that you remove? **$47x^2 - 12x + 1$ cubic units**

34. What are the zeros and the multiplicity of each zero for the polynomial function $x^4 - 2x^2 + 1$? **1 of multiplicity 2, −1 of multiplicity 2**

35. Error Analysis On your homework, you wrote that the polynomial function from the given zeros $x = 3, 0, -9, 1$ is $y = x^4 + 5x^3 - 33x^2 + 27x$. Your friend wrote that the polynomial function is $y = x^3 + 5x^2 - 33x + 27$. Who is correct? What mistake was made?
You are correct. Your friend did not include the factor with the zero $x = 0$ from the polynomial. It should include the factor $(x - 0)$.

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5-2 Practice

Form K

Polynomials, Linear Factors, and Zeros

Write each polynomial in factored form. Check by multiplication.

1. $x^3 + 11x^2 + 30x$

To start, factor out the GCF, x. $\quad x(x^2 + 11x + 30)$

$x(x + 5)(x + 6)$

2. $x^3 - 3x^2 - x + 3$

$(x - 1)(x + 1)(x - 3)$

3. $x^2 - 4x - 12$

$(x + 2)(x - 6)$

4. $x^3 - 81x$

$x(x + 9)(x - 9)$

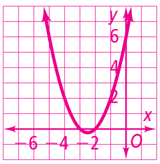
5. $x^3 + 9x^2 + 18x$

$x(x + 6)(x + 3)$

Find the zeros of each function. Then graph the function.

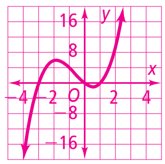
6. $y = (x + 2)(x + 3)$

$-2, -3$



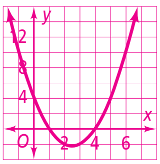
7. $y = x(x - 1)(x + 3)$

0, 1, -3



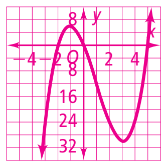
8. $y = (x - 4)(x - 1)$

4, 1



9. $y = x(x - 5)(x + 2)$

0, 5, -2



Write a polynomial function in standard form with the given zeros.

10. $x = -2, 1, 4$

To start, write a linear factor for each zero. $\quad (x - (-2))(x - 1)(x - 4)$

Simplify $\quad (x + 2)(x - 1)(x - 4)$

$y = x^3 - 3x^2 - 6x + 8$

11. $x = 3, 0$

$y = x^2 - 3x$

12. $3, -8, 0$

$y = x^3 + 5x^2 - 24x$

13. $x = 3, -2, 1$

$y = x^3 - 2x^2 - 5x + 6$

14. $x = -4, 1$

$y = x^2 + 3x - 4$

Name ______________________ Class ____________ Date ____________

5-2 Practice (continued) *Form K*

Polynomials, Linear Factors, and Zeros

Find the zeros of each function. State the multiplicity of multiple zeros.

15. $y = (x - 3)^2(x + 1)$

To start, identify the zeros. The zeros are 3 and -1.

3 multiplicity 2, −1 multiplicity 1

16. $y = x^2 + 3x + 2$

−1 multiplicity 1, −2 multiplicity 1

17. $y = (x + 5)^2$

−5 multiplicity 2

18. $y = (x - 9)^2$

9 multiplicity 2

19. $y = 2x^2 - 2x$

0 multiplicity 1, 1 multiplicity 1

Find the relative maximum and relative minimum of the graph of each function.

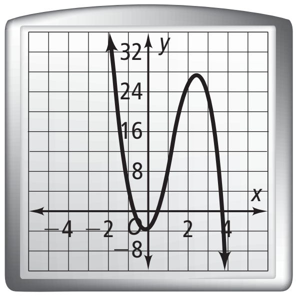
20. $f(x) = -3x^3 + 10x^2 + 6x - 3$

To start, use a graphing calculator.

(An approximate viewing window is

$-5 \leq x \leq 5$ and $-10 \leq y \leq 30$.)

rel max at (2.5, 27.6), rel min at (−0.3, −3.8)



21. $f(x) = x^3 + 4x^2 - x + 1$

rel max at (−2.8, 13.2), rel min at (0.1, 0.9)

22. $f(x) = x^3 - 6x + 9$

rel max at (−1.4, 14.7), rel min at (1.4, 3.3)

23. Reasoning A polynomial function has a zero at $x = b$. Find one of its factors.

$(x - b)$

24. The side of a cube measures $2x + 1$ units long. Express the volume of the cube as a polynomial.

$8x^3 + 12x^2 + 6x + 1$

25. The length of a box is 2 times the height. The sum of the length, width, and height of the box is 10 centimeters.

a. Write expressions for the dimensions of the box. **length: $2x$; width: $10 - 3x$; height: x**

b. Write a polynomial function for the volume of the box. (To start, write the function in factored form). **$f(x) = 20x^2 - 6x^3$**

c. Find the maximum volume of the box and the dimensions of the box that generates this volume.

max vol: 32.9 cm^3; length: 4.4 cm; width: 3.4 cm; height: 2.2 cm

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5-2 Standardized Test Prep

Polynomials, Linear Factors, and Zeros

Multiple Choice

For Exercises 1–6, choose the correct letter.

1. What are the zeros of the polynomial function $y = (x - 3)(2x + 1)(x - 1)$? C

(A) $\frac{1}{2}, 1, 3$ (B) $-1, 1, 3$ (C) $-\frac{1}{2}, 1, 3$ (D) $-3, \frac{1}{2}, -1$

2. What is the factored form of $2x^3 + 5x^2 - 12x$? H

(F) $(x + 4)(2x - 3)$ (H) $x(x + 4)(2x - 3)$

(G) $(x - 4)(2x + 3)$ (I) $x(x - 4)(2x + 3)$

3. Which is the cubic polynomial in standard form with roots 3, -6, and 0? D

(A) $x^2 - 3x - 18$ (C) $x^3 - 3x^2 - 18x$

(B) $x^2 + 3x - 18$ (D) $x^3 + 3x^2 - 18x$

4. What is the relative minimum and relative maximum of $f(x) = 6x^3 - 5x + 12$? I

(F) min $= 0$, max $= 0$ (H) min $= -1.5$, max $= 12$

(G) min $= -5$, max $= 6$ (I) min $= 10.2$, max $= 13.8$

5. What is the multiplicity of the zero of the polynomial function $f(x) = (x + 5)^4$? A

(A) 4 (B) 5 (C) 20 (D) 625

6. For the polynomial function $y = (x - 2)^3$, how does the graph behave at the x-intercept? H

(F) linear (G) quadratic (H) cubic (I) quartic

Short Response

7. A rectangular box is 24 in. long, 12 in. wide, and 18 in. high. If each dimension is increased by x in., what is the polynomial function in standard form that models the volume V of the box? Show your work.

[2] $V = x^3 + 54x^2 + 936x + 5184$ in.^{3}

[1] correct process with one computational error OR correct polynomial without work shown

[0] incorrect answer and no work shown OR no answer given

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5-2 Enrichment

Polynomials, Linear Factors, and Zeros

Fast Factorization of Monic Trinomials

Thus far, you have factored trinomials primarily by trial and error. This method can be quite slow, especially if the constant term of a quadratic trinomial is, for example, 72, since there are many different ways to factor this number: 1 and 72, 2 and 36, 3 and 24, 4 and 18, 6 and 12, 8 and 9. With larger numbers, the trial-and-error method becomes time-consuming.

A polynomial is called monic if the coefficient of the term of highest degree is 1. Monic quadratic trinomials can be factored quite rapidly using a combination of difference of squares and equation solving.

Suppose that we wish to factor the monic quadratic trinomial $x^2 + bx + c$. Assume there are numbers r and s such that

$$\begin{aligned} x^2 + bx + c &= [(x + r) + s][(x + r) - s] \\ &= (x + r)^2 - s^2 \\ &= x^2 + 2rx + r^2 - s^2 \end{aligned}$$

Subtracting x^2 from both sides results in

$$bx + c = 2rx + r^2 - s^2$$

Therefore, $b = 2r$ and $c = r^2 - s^2$.

Solve for r and s in terms of b and c.

$$r = \frac{b}{2} \qquad s = \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

The following example shows how this technique works.

Factor: $x^2 - 6x - 72$ $\qquad b = -6, c = -72$

$r = \frac{b}{2} = -3$ $\qquad s = \sqrt{9 - (-72)}$

$\qquad = \sqrt{81} = 9$

So $x^2 - 6x - 72$ $\qquad = [(x - 3) + 9][(x - 3) - 9)]$

$\qquad = (x + 6)(x - 12)$

Verify that this is the correct factorization.

$(x + 6)(x - 12) = x^2 - 12x + 6x - 72 = x^2 - 6x - 72$

Use this technique to factor each of the following.

1. $x^2 - 60x + 864$ **$(x - 24)(x - 36)$**

2. $x^2 - 25x + 144$ **$(x - 9)(x - 16)$**

3. $x^2 - 28x - 128$ **$(x - 32)(x + 4)$**

4. $x^2 - 44x - 540$ **$(x - 54)(x + 10)$**

5. $x^2 + 81x + 990$ **$(x + 15)(x + 66)$**

6. $x^2 - 50x + 576$ **$(x - 32)(x - 18)$**

7. $x^2 + 17x - 168$ **$(x - 7)(x + 24)$**

8. $x^2 - 17x - 630$ **$(x - 35)(x + 18)$**

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5-2 Reteaching

Polynomials, Linear Factors, and Zeros

The Factor Theorem tells you that if you know the zeros of a polynomial function, you can write the polynomial.

Factor Theorem

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Problem

What is a cubic polynomial function in standard form with zeros 0, 4, and -2?

Each zero (a) is part of a linear factor of the polynomial, so you can write each factor as $(x - a)$.

$(x - a_1)(x - a_2)(x - a_3)$	Set up the cubic polynomial factors.
$a_1 = 0, a_2 = 4, a_3 = -2$	Assign the zeros.
$(x - 0)(x - 4)[x - (-2)]$	Substitute the zeros into the factors.
$f(x) = x(x - 4)(x + 2)$	Write the polynomial function in factored form.
$f(x) = x(x^2 - 2x - 8)$	Multiply $(x - 4)(x + 2)$.
$f(x) = x^3 - 2x^2 - 8x$	Multiply by x using the Distributive Property.

The polynomial function written in standard form is $f(x) = x^3 - 2x^2 - 8x$.

Exercises

Write a polynomial function in standard form with the given zeros.

1. $5, -1, 3$ $f(x) = x^3 - 7x^2 + 7x + 15$

2. $1, 7, -5$ $f(x) = x^3 - 3x^2 - 33x + 35$

3. $-1, 1, -6$ $f(x) = x^3 + 6x^2 - x - 6$

4. $2, -2, -3$ $f(x) = x^3 + 3x^2 - 4x - 12$

5. $2, 1, 3$ $f(x) = x^3 - 6x^2 + 11x - 6$

6. $2, 3, -3, -1$ $f(x) = x^4 - x^3 - 11x^2 + 9x + 18$

7. $0, -8, 2$ $f(x) = x^3 + 6x^2 - 16x$

8. $-10, 0, 2$ $f(x) = x^3 + 8x^2 - 20x$

9. $-2, 2, -\frac{3}{2}$ $f(x) = x^3 + \frac{3}{2}x^2 - 4x - 6$

10. $-1, \frac{2}{3}$ $f(x) = x^2 + \frac{1}{3}x - \frac{2}{3}$

5-2 Reteaching (continued)

Polynomials, Linear Factors, and Zeros

You can use a polynomial function to find the minimum or maximum value of a function that satisfies a given set of conditions.

Problem

Your school wants to put in a swimming pool. The school wants to maximize the volume while keeping the sum of the dimensions at 40 ft. If the length must be 2 times the width, what should each dimension be?

Step 1 First, define a variable x. Let $x =$ the width of the pool.

Step 2 Determine the length and depth of the pool using the information in the problem.

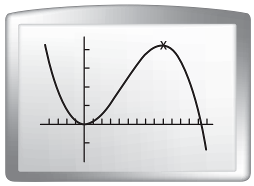
The length must be 2 times the width, so length $= 2x$.

The length plus width plus depth must equal 40 ft, so depth $= 40 - x - 2x = 40 - 3x$.

Step 3 Create a polynomial in standard form using the volume formula

$$V = \text{length} \cdot \text{width} \cdot \text{depth}$$
$$= 2x(x)(40 - 3x)$$
$$= -6x^3 + 80x^2$$

Step 4 Graph the polynomial function. Use the MAXIMUM feature. The maximum volume is 2107 ft^3 at a width of 8.9 ft.



Maximum
X = 8.8888882 Y = 2106.9959

Step 5 Evaluate the remaining dimensions: width $= x \approx 8.9$ ft

length $= 2x \approx 17.8$ ft

depth $= 40 - 3x \approx 13.3$ ft

Exercises

11. Find the dimensions of the swimming pool if the sum must be 50 ft and the length must be 3 times the depth. **depth = 8.3 ft, length = 24.9 ft, width = 16.8 ft**

12. Find the dimensions of the swimming pool if the sum must be 40 ft and the depth must be one tenth of the length. **length = 24.2 ft, depth = 2.42 ft, width = 13.4 ft**

13. Find the dimensions of the swimming pool if the sum must be 60 ft and the length and width are equal. **length = 20 ft, width = 20 ft, depth = 20 ft**

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5-3 Additional Vocabulary Support

Solving Polynomial Equations

Problem

What are the real or imaginary solutions of the equation $4x^3 + 2x^2 = 2x$? Explain your work.

$4x^3 + 2x^2 = 2x$	Write the original equation.
$4x^3 + 2x^2 - 2x = 0$	Rewrite in the form $P(x) = 0$.
$2x^3 + x^2 - x = 0$	Multiply by $\frac{1}{2}$ to simplify.
$x(2x^2 + x - 1) = 0$	Factor out the GCF, x.
$x(2x - 1)(x + 1) = 0$	Factor $2x^2 + x - 1$.
$x = 0$ or $2x - 1 = 0$ or $x + 1 = 0$	Zero Product Property
$x = 0 \quad x = \frac{1}{2} \quad x = -1$	Solve each equation for x.

Exercises

What are the real or imaginary solutions of the equation $2x^3 + x^2 = 6x$? Explain your work.

$2x^3 + x^2 = 6x$	**Write the original equation.**
$2x^3 + x^2 - 6x = 0$	**Rewrite in the form $P(x) = 0$.**
$x(2x^2 + x - 6) = 0$	**Factor out the GCF, x.**
$x(2x - 3)(x + 2) = 0$	**Factor $2x^2 + x - 6$.**
$x = 0$ or $2x - 3 = 0$ or $x + 2 = 0$	**Zero Product Property**
$x = 0 \quad x = \frac{3}{2} \quad x = -2$	**Solve each equation for x.**

What are the real or imaginary solutions of the equation $3x^3 + 7x^2 = 6x$? Explain your work.

$3x^3 + 7x^2 = 6x$	**Write the original equation.**
$3x^3 + 7x^2 - 6x = 0$	**Rewrite in the form $P(x) = 0$.**
$x(3x^2 + 7x - 6) = 0$	**Factor out the GCF, x.**
$x(3x - 2)(x + 3) = 0$	**Factor $3x^2 + 7x - 6$.**
$x = 0, 3x - 2 = 0, x + 3 = 0$	**Zero Product Property**
$x = 0, x = \frac{2}{3}, x = -3$	**Solve each equation for x.**

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5-3 Think About a Plan

Solving Polynomial Equations

Geometry The width of a box is 2 m less than the length. The height is 1 m less than the length. The volume is 60 m^3. What is the length of the box?

Know

1. The volume of the box is $\boxed{60 \text{ m}^3}$.
2. The formula for the volume of a rectangular prism is $\boxed{V = l \times w \times h}$.
3. The width of the box is equal to $\boxed{\text{the length}}$ − 2 m.
4. The height of the box is equal to $\boxed{\text{the length}}$ − 1 m.

Need

5. To solve the problem I need to:
   write an equation expressing the volume of the box two ways and solve the equation for the length of the box.

Plan

6. Define a variable. Let x = $\boxed{\text{length}}$.
7. What variable expressions represent the width and height of the box?
   $\boxed{x - 2}$ and $\boxed{x - 1}$
8. What equation expresses the volume of the box in two ways? $x(x - 2)(x - 1) = 60$
9. How can you use a graphing calculator to help you solve the equation?
   Answers may vary. Sample: Change the equation to $P(x) = 0$ form. Then graph $P(x)$ on a graphing calculator. Use the zero feature to find real solutions.
10. What is the solution of the equation? $x = 5$
11. What are the dimensions of the box? Are the solutions reasonable?
    The length is 5 m, the width is 3 m, and the height is 4 m. All are reasonable dimensions for a box.

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5-3 Practice — Form G

Solving Polynomial Equations

Find the real or imaginary solutions of each equation by factoring.

1. $8x^3 - 27 = 0$
$(2x - 3)(4x^2 + 6x + 9); \frac{3}{2}, \frac{-3 \pm 3i\sqrt{3}}{4}$

2. $x^3 + 64 = 0$
$(x + 4)(x^2 - 4x + 16); -4, 2 \pm 2i\sqrt{3}$

3. $2x^3 + 54 = 0$
$2(x + 3)(x^2 - 3x + 9); -3, \frac{3 \pm 3i\sqrt{3}}{2}$

4. $2x^3 - 250 = 0$
$2(x - 5)(x^2 + 5x + 25); 5, \frac{-5 \pm 5i\sqrt{3}}{2}$

5. $4x^3 - 32 = 0$
$4(x - 2)(x^2 + 2x + 4); 2, -1 \pm i\sqrt{3}$

6. $27x^3 + 1 = 0$
$(3x + 1)(9x^2 - 3x + 1); -\frac{1}{3}, \frac{1 \pm i\sqrt{3}}{6}$

7. $64x^3 - 1 = 0$
$(4x - 1)(16x^2 + 4x + 1); \frac{1}{4}, \frac{-1 \pm i\sqrt{3}}{8}$

8. $x^3 - 27 = 0$
$(x - 3)(x^2 + 3x + 9); 3, \frac{-3 \pm 3i\sqrt{3}}{2}$

9. $x^4 - 5x^2 + 4 = 0$
$(x + 1)(x - 1)(x + 2)(x - 2); -2, -1, 1, 2$

10. $x^4 - 12x^2 + 11 = 0$
$(x + 1)(x - 1)(x^2 - 11); -1, 1, -\sqrt{11}, \sqrt{11}$

11. $x^4 - 10x^2 + 16 = 0$
$(x^2 - 2)(x^2 - 8); -\sqrt{2}, \sqrt{2}; -2\sqrt{2}, 2\sqrt{2}$

12. $x^4 - 8x^2 + 16 = 0$
$(x + 2)^2(x - 2)^2; -2, 2$

13. $x^4 - 9x^2 + 14 = 0$
$(x^2 - 7)(x^2 - 2); -\sqrt{7}, \sqrt{7}, -\sqrt{2}, \sqrt{2}$

14. $x^4 + 13x^2 + 36 = 0$
$(x^2 + 4)(x^2 + 9); -2i, 2i, -3i, 3i$

15. $x^4 - 10x^2 + 9 = 0$
$(x + 1)(x - 1)(x + 3)(x - 3); -1, 1, -3, 3$

16. $x^4 + 3x^2 - 4 = 0$
$(x + 1)(x - 1)(x^2 + 4); -1, 1, -2i, 2i$

Find the real solutions of each equation by graphing.

17. $2x^4 = 9x^2 - 4$ $-2, 2, -0.71, 0.71$

18. $x^2 - 16x = -1$ $0.06, 15.94$

19. $6x^3 + 10x^2 + 5x = 0$ 0

20. $36x^3 + 6x^2 = 9x$ $-0.59, 0, 0.42$

21. $15x^4 = 11x^3 + 14x^2$ $-0.67, 0, 1.4$

22. $x^4 = 81x^2$ $-9, 0, 9$

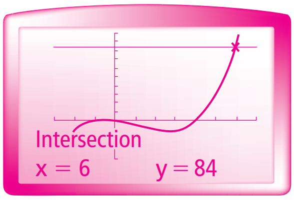
For Exercises 23 and 24, write an equation to model each situation. Then solve each equation by graphing.

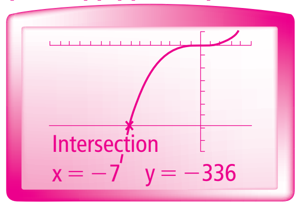
23. The volume V of a container is 84 ft^3. The width, the length, and the height are x, $x + 1$, and $x - 4$ respectively. What are the container's dimensions?

24. The product of three consecutives integers $n - 1$, n, and $n + 1$ is -336. What are the integers?

24. $(n - 1)(n)(n + 1) = -336; -8, -7, -6$

23. $x^3 - 3x^2 - 4x = 84$
$x = 6$ ft
$x + 1 = 7$ ft
$x + 4 = 2$ ft





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5-3 Practice (continued) — Form G

Solving Polynomial Equations

Solve each equation.

25. $x^4 - x = 0$
$0, 1, \frac{-1 \pm i\sqrt{3}}{2}$

26. $3x^4 + 18 = 21x^2$
$-1, 1, -\sqrt{6}, \sqrt{6}$

27. $2x^4 - 26x^2 - 28 = 0$
$-\sqrt{14}, \sqrt{14}, -i, i$

28. $5x^4 + 50x^2 + 80 = 0$
$-i\sqrt{2}, i\sqrt{2}, -2i\sqrt{2}, 2i\sqrt{2}$

29. $x^4 - 81 = 0$
$-3, 3, -3i, 3i$

30. $x^4 = 25$
$-\sqrt{5}, \sqrt{5}, -i\sqrt{5}, i\sqrt{5}$

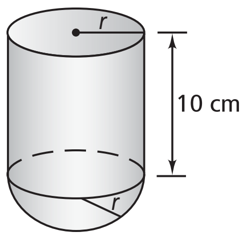
31. $x^5 = x^3 + 12x$
$0, -2, 2, -i\sqrt{3}, i\sqrt{3}$

32. $x^4 + 12x^2 = 8x^3$
$0, 2, 6$

33. Over 3 years, you save your earnings from a summer job. The polynomial $1600x^3 + 1200x^2 + 800x$ represents your savings, with interest, at the end of the 3 years. The annual interest rate equals $x - 1$. Find the interest rate needed so that you will have $4000 at the end of 3 years. **4.82%**

34. Error Analysis Your friend claims that the zeros of $3x^3 + 7x^2 - 22x - 8 = 0$ are -4, 2, and -1. What did your friend do wrong? What are the correct factors? **Your friend forgot to divide by 3 when solving an equation to find the third factor. The correct factors are -4, 2, and $-\frac{1}{3}$.**

35. The container at the right consists of a cylinder on top of a hemisphere. The container holds 500 cm^3. What is the radius of the container, to the nearest hundredth of a centimeter? **3.58 cm**



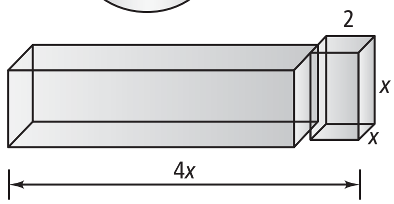
36. Suppose a 2-in. slice is cut from one face of the cheese block as shown. The remaining block has a volume of 224 $in.^3$.

a. What are the dimensions of the new block? **4 in. × 4 in. × 14 in.**

b. What are the dimensions of the old block? **4 in. × 4 in. × 16 in.**

c. What is the original volume? **256 $in.^3$**

d. What is the volume of the cut slice? **32 $in.^3$**



37. Reasoning A test question asks you to find three integers whose product is 412. Do you have enough information to solve this problem? Explain.
Yes, but there are multiple solutions.

38. Your mother is 25 years older than you. Your father is 3 years older than your mother. The product of all three ages is 32,130. How old is your father? **45 years old**

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5-3 Practice

Form K

Solving Polynomial Equations

Find the real or imaginary solutions of each equation by factoring.

1. $x^3 + 512 = 0$

To start, write $x^3 + 512$ as a sum of cubes and factor.

$x^3 + 8^3 = 0$

$(x + 8)(x^2 - 8x + 64) = 0$

$x = -8, 4 \pm 4i\sqrt{3}$

2. $x^4 - 3x^2 = -2x^2$

$x = 0, 1, -1$

3. $x^4 + 5x^2 = 6$

$x = \pm 1, \pm i\sqrt{6}$

4. $x^4 + 2x^3 = 10x^2$

$x = 0, -1 \pm \sqrt{11}$

5. $27x^3 - 1 = 0$

$x = \frac{1}{3}, \frac{-1 \pm i\sqrt{3}}{6}$

6. $x^2 + 4 = -4x$

$x = -2$

7. $x^3 + 10x^2 + 24x = 0$

$x = 0, -4, -6$

Solve each equation.

8. $x^3 + 4x^2 - 2x - 8 = 0$

$x = -4, \pm\sqrt{2}$

9. $x^5 - 6x^3 + 5x = 0$

$x = 0, \pm 1, \pm\sqrt{5}$

10. $x^3 = 9x$

$x = 0, 3, -3$

11. $2x^3 + 8x^2 + 4x = -16$

$x = -4, \pm i\sqrt{2}$

12. $x^4 - 25 = 0$

$x = \pm\sqrt{5}, \pm i\sqrt{5}$

13. $27x^3 - 216 = 0$

$x = 2, -1 \pm i\sqrt{3}$

14. Writing Show how you can rewrite $\frac{x^6}{y^9} - \frac{1}{27}$ as a difference of two cubes.

$$\frac{x^6}{y^9} - \frac{1}{27} = \left(\left(\frac{x^2}{x^3}\right)^3 - \left(\frac{1}{3}\right)^3\right)$$

5-3 Practice (continued) *Form K*

Solving Polynomial Equations

Find the real solutions of each equation by graphing.

15. $x^3 - 3x^2 - 9x = -15$

To start, rewrite the equation with one side equal to zero. $x^3 - 3x^2 - 9x + 15 = 0$

$x = -2.62, 1.34, 4.28$

16. $3x^2 = 22x$
$x = 0, 7.33$

17. $2x^4 - 2x^3 + 4x^2 = 3$
$x = -0.69, 0.89$

18. $-x^3 + 1 = 2x^2$
$x = -1.62, -1, 0.62$

19. $x^4 - 2x^3 - 5x^2 + 1 = 0$
$x = -1.33, -0.52, 0.42, 3.43$

20. $2x^2 = -6x$
$x = -3, 0$

21. $x^3 + 5x^2 = 9x$
$x = -6.41, 0, 1.41$

For Exercises 22–25, write an equation to model each situation. Then solve each equation by graphing.

22. The volume V of a container is 61 in.^{3}. The width, the length, and the height are x, $x - 2$, and $x + 3$ respectively. What are the container's dimensions?

To start, write the equation of the volume of the container.

$x(x - 2)(x + 3) = 61$

$x^3 + x^2 - 6x = 61$

width: 4.10 in.; length: 2.10 in.; height: 7.10 in.

23. The product of three consecutive integers is 720. What are the numbers?
$x^3 + 3x^2 + 2x - 720 = 0$; **the numbers are 8, 9 and 10.**

24. The height of a box is 3 cm less than the width. The length is 2 cm less than the width. The volume is 50 cm^3. What is the width of the box?
$x^3 - 5x^2 + 6x - 50 = 0$; **the width is 5.54 cm**

25. Your sister is 8 years older than you. Your mother is 25 years older than your sister. The product of all three ages is 18,816. How old is your mother?
$x^3 + 41x^2 + 264x - 18{,}816 = 0$; **your mother is 49 years old.**

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5-3 Standardized Test Prep

Solving Polynomial Equations

Multiple Choice

For Exercises 1–6, choose the correct letter.

1. If you factor $x^3 - 8$ in the form $(x - a)(x^2 + bx + c)$, what is the value of a? **A**

(A) 2 (B) -2 (C) 4 (D) -4

2. The product of three integers x, $x + 2$, and $x - 5$ is 240. What are the integers? **I**

(F) 5.9, 7.9, 0.9 (G) 7.5, 9.5, 2.5 (H) 5, 6, 8 (I) 8, 10, 3

3. Over 3 years, you save \$550, \$600, and \$650 from babysitting jobs. The polynomial $550x^3 + 600x^2 + 650x$ represents your total bank account balance after 3 years. The annual interest rate is $x - 1$. What is the interest rate needed so that you will have \$2000 after 3 years? **C**

(A) 0.06% (B) 1.06% (C) 5.52% (D) 24%

4. Which polynomial equation has the zeros 5, -3, and $\frac{1}{2}$? **H**

(F) $x^3 + 4x^2 + 4x - 45$

(G) $x^3 - 4x^2 + 4x + 15$

(H) $2x^3 - 5x^2 - 28x + 15$

(I) $2x^3 + 5x^2 - 28x - 45$

5. Your brother is 3 years older than you. Your sister is 4 years younger than you. The product of your ages is 1872. How old is your sister? **A**

(A) 9 years (B) 13 years (C) 16 years (D) 17 years

6. What are the real roots of $x^3 + 8 = 0$? **G**

(F) 2 (G) -2 (H) $-2 \pm \sqrt{3}$ (I) $-2 \pm \sqrt{5}$

Short Response

7. You have a block of wood with a depth of x units, a length of $5x$ units, and a height of $2x$ units. You need to cut a slice off the top of the block to decrease the height by 2 units. The new block will have a volume of 480 cubic units.

a. What are the dimensions of the new block?

b. What is the volume of the slice?

[2] a. 4 units deep, 20 units long, 6 units high; b. 160 cubic units
[1] incorrect dimensions OR incorrect slice volume
[0] no answers given

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5-3 Enrichment

Solving Polynomial Equations

Factoring $x^2 + a^2$

There is a formula for factoring $x^2 - a^2$, a formula for factoring $x^3 - a^3$, and a formula for factoring $x^3 + a^3$. Why isn't there a formula for factoring $x^2 + a^2$?

Since $x^2 + a^2$ must be the product of two first-degree polynomials, assume that $x^2 + a^2 = (x + b)(x + c)$. Use the distributive property to expand the expression on the right.

$$x^2 + a^2 = (x + b)(x + c)$$

$$x^2 + a^2 = x^2 + bx + cx + bc$$

$$a^2 = (b + c)x + bc \qquad \text{Subtract } x^2 \text{ from each side.}$$

How can you tell whether there are real values of a, b, and c that will make this equation true for all x? Look at each side of the equation as a linear function.

$$y = a^2$$

$$y = (b + c)x + bc$$

If these two functions are the same, they must have the same graph, the same slope, and the same y-intercept.

1. Find the slope of $y = a^2$. **0**

2. Find the slope of $y = (b + c)x + bc$. **$b + c$**

3. What relationship must exist between b and c? **$b = -c$**

4. **a.** Find the y-intercept of $y = a^2$. **a^2**
b. Find the y-intercept of $y = (b + c)x + bc$. **bc**
c. What relationship must exist among a, b, and c? **$a^2 = -b^2$, $a^2 = -c^2$**

5. Are there real nonzero values of a, b, and c that can make the original equation $x^2 + a^2 = (x + b)(x + c)$ true? Explain.
No; a^2 will always be a positive number, and $-b^2$ and $-c^2$ will always be negative numbers.

This type of proof is called an indirect proof. To prove that something is not true, you begin by assuming that it is true and follow through systematically until you reach a contradiction.

6. What are the complex factors of $x^2 + a^2$? **$(x + ai)(x - ai)$**

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5-3 Reteaching

Solving Polynomial Equations

Problem

What are the real or imaginary solutions of the polynomial equation $2x^3 + 16 = 0$?

$2x^3 + 16 = 0$	
$2(x^3 + 8) = 0$	Factor out the GCF. In this case, it is 2.
$2(x + 2)(x^2 - 2x + 4) = 0$	Factor the remaining cubic expression.
$x + 2 = 0$ or $x^2 - 2x + 4 = 0$	Use the Zero-Product Property.
$x = -2$ or $x = \dfrac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$	Solve each equation for x. Use the Quadratic Formula when necessary.
$x = -2$ or $x = \dfrac{2 \pm 2i\sqrt{3}}{2}$	
$x = -2$ or $x = 1 \pm i\sqrt{3}$	Simplify.

The solutions are -2 and $1 \pm i\sqrt{3}$.

Exercises

Find the real or imaginary solutions of each polynomial equation.

1. $x^3 - 8 = 0$
$2, -1 \pm i\sqrt{3}$

2. $4x^3 + 4 = 0$
$-1, \frac{1 \pm i\sqrt{3}}{2}$

3. $x^4 - x^2 - 72 = 0$
$-3, 3, -2i\sqrt{2}, 2i\sqrt{2}$

4. $x^4 + 9x^2 = -20$
$-2i, 2i, -i\sqrt{5}, i\sqrt{5}$

5. $x^4 - 27x = 0$
$0, 3, \frac{-3 \pm 3i\sqrt{3}}{2}$

6. $8x^3 = -1$
$-\frac{1}{2}, \frac{1 \pm i\sqrt{3}}{4}$

7. $7x^4 = -28x^2 - 21$
$-i, i, -i\sqrt{3}, i\sqrt{3}$

8. $x^3 = 64$
$4, -2 \pm 2i\sqrt{3}$

9. $8x^3 + 27 = 0$
$-\frac{3}{2}, \frac{3 \pm 3i\sqrt{3}}{4}$

10. $x^4 - 7x^2 = -12$
$-2, 2, -\sqrt{3}, \sqrt{3}$

11. $2x^4 + 16x^2 = 40$
$-\sqrt{2}, \sqrt{2}, -i\sqrt{10}, i\sqrt{10}$

12. $2x^4 - 16x = 0$
$0, 2, -1 \pm i\sqrt{3}$

13. $9x^4 - 25 = 0$
$-\frac{\sqrt{15}}{3}, \frac{\sqrt{15}}{3}, -i\frac{\sqrt{15}}{3}, i\frac{\sqrt{15}}{3}$

14. $2x^4 - x^2 = 3$
$-\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}, -i, i$

15. $x^4 + 5x^2 = -4$
$-i, i, -2i, 2i$

16. $x^4 - 7x^2 - 8 = 0$
$-2\sqrt{2}, 2\sqrt{2}, -i, i$

17. $2x^3 + 16 = 0$
$-2, 1 \pm i\sqrt{3}$

18. $x^4 - 5x^2 - 24 = 0$
$-2\sqrt{2}, 2\sqrt{2}, -i\sqrt{3}, i\sqrt{3}$

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5-3 Reteaching (continued)

Solving Polynomial Equations

Problem

You have a brother and a sister. Your brother is 3 years older than you. Your sister is 2 years older than your brother. The product of all three ages is 3744. How old are you and your siblings?

Step 1 Define the variables.

Let your age $= x$.
Your brother's age $= x + 3$.
Your sister's age $= x + 3 + 2 = x + 5$.

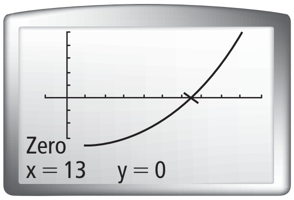
Step 2 Write an equation.

Because the product of all three ages equals 3744, $x(x + 3)(x + 5) = 3744$.

Step 3 Rewrite the equation in $P(x) = 0$ form.

$(x^2 + 3x)(x + 5) = 3744$
$x^3 + 3x^2 + 5x^2 + 15x = 3744$
$x^3 + 8x^2 + 15x - 3744 = 0$

Step 4 Using a graphing calculator, graph the equation.
Use the Zero feature to solve for x.



Step 5 Once you have the value of x (your age), you can solve for the other ages.
Since $x = 13$, your brother's age is $x + 3 = 16$.
Your sister's age is $x + 5 = 18$.

Exercises

19. A slice of wood 3 in. thick is cut off a cube of wood. The remaining solid has a volume of 320 in.^{2}. What are the dimensions of the original block of wood?
8 in. × 8 in. × 8 in.

20. The water level in a rectangular fish tank is 4 in. from the top. The depth of the water is the same as the width of the tank, which is half of its length. The volume of the water in the tank is 4394 in.^{3}. What is the volume of the fish tank?
5746 in.^{3}

5-4 Additional Vocabulary Support

Dividing Polynomials

The column on the left shows the steps used to divide one polynomial by another using polynomial long division. Use the column on the left to answer the questions in the column on the right.

Problem **Using Polynomial Long Division**

Use polynomial long division to divide $6x^2 + x - 12$ by $2x + 3$. What is the quotient?

1. How is polynomial long division similar to standard long division?

Answers may vary. Sample: Both polynomial long division and standard long division involve dividing a divisor into a dividend to find a quotient. A division house is used in both cases, and they both involve bringing numbers down and subtracting.

Use the polynomials to write a long division problem.

$$2x + 3\overline{)6x^2 + x - 12}$$

2. Which polynomial is the divisor? Which is the dividend?

$2x + 3$; $6x^2 + x - 12$

Divide $2x$ into $6x^2$.

$$\begin{array}{r} 3x \phantom{{}+x-12} \\ 2x + 3\overline{)6x^2 + x - 12} \end{array}$$

3. Why is $3x$ rather than $3x^2$ the first term in the quotient?

The product of $3x^2$ and $2x$ is $6x^3$, not $6x^2$.

Multiply $3x(2x + 3)$ and subtract.

$$\begin{array}{r} 3x \phantom{{}+x-12} \\ 2x + 3\overline{)6x^2 + x - 12} \\ \underline{6x^2 + 9x} \phantom{{}-12} \\ -8x \phantom{{}-12} \end{array}$$

4. Which property do you use when you multiply $3x(2x + 3)$?

Distributive Property

Divide $2x$ into $-8x$.

$$\begin{array}{r} 3x - 4 \phantom{{}-12} \\ 2x + 3\overline{)6x^2 + x - 12} \\ \underline{6x^2 + 9x} \phantom{{}-12} \\ -8x - 12 \end{array}$$

5. How would the quotient be affected if $2x + 3$ is changed to $-2x + 3$?

The quotient would change from -4 to 4.

Multiply $-4(2x + 3)$ and subtract.

$$\begin{array}{r} 3x - 4 \phantom{{}-12} \\ 2x + 3\overline{)6x^2 + x - 12} \\ \underline{6x^2 + 9x} \phantom{{}-12} \\ -8x - 12 \\ \underline{-8x - 12} \\ 0 \end{array}$$

6. Subtracting a negative term is equivalent to performing what operation to the opposite of the negative term?

addition

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5-4 Think About a Plan

Dividing Polynomials

Geometry The expression $\frac{1}{3}(x^3 + 5x^2 + 8x + 4)$ represents the volume of a square pyramid. The expression $x + 1$ represents the height of the pyramid. What expression represents the side length of the base? (*Hint:* The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$.)

Understanding the Problem

1. What expression represents the height of the pyramid? **$x + 1$**

2. What does B represent in the formula for the volume of a pyramid?

the area of the base of the pyramid

3. What is the problem asking you to determine?

the side length of the base of the pyramid

Planning the Solution

4. How can you find an expression that represents B?

Factor $\frac{1}{3}$ and $(x + 1)$ out of the expression for the volume. The remaining expression represents B.

5. How can polynomial division help you solve this problem?

Divide the expression for the volume by $(x + 1)$ to find another factor of the expression.

6. How can you find the side length of the base once you find an expression for B?

The base is square because the pyramid is a square pyramid. The side length is $\sqrt{B}$.

Getting an Answer

7. What expression represents B, the area of the base? **$x^2 + 4x + 4$**

8. What expression represents the side length of the base? **$x + 2$**

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5-4 Practice

Form G

Dividing Polynomials

Divide using long division. Check your answers.

1. $(x^2 - 13x - 48) \div (x + 3)$
$x - 16$

2. $(2x^2 + x - 7) \div (x - 5)$
$2x + 11$, R 48

3. $(x^3 + 5x^2 - 3x - 1) \div (x - 1)$
$x^2 + 6x + 3$, R 2

4. $(3x^3 - x^2 - 7x + 6) \div (x + 2)$
$3x^2 - 7x + 7$, R -8

5. $(x^2 - 3x + 1) \div (x - 4)$
$x + 1$, R 5

6. $(x^3 - 4x^2 + 3x + 2) \div (x + 2)$
$x^2 - 6x + 15$, R -28

Determine whether each binomial is a factor of $x^3 + 3x^2 - 10x - 24$.

7. $x + 4$ **yes**

8. $x - 3$ **yes**

9. $x + 6$ **no**

10. $x + 2$ **yes**

Divide using synthetic division.

11. $(x^3 - 8x^2 + 17x - 10) \div (x - 5)$
$x^2 - 3x + 2$

12. $(x^3 + 5x^2 - x - 9) \div (x + 2)$
$x^2 + 3x - 7$, R 5

13. $(-2x^3 + 15x^2 - 22x - 15) \div (x - 3)$
$-2x^2 + 9x + 5$

14. $(x^3 + 7x^2 + 15x + 9) \div (x + 1)$
$x^2 + 6x + 9$

15. $(x^3 + 2x^2 + 5x + 12) \div (x + 3)$
$x^2 - x + 8$, R -12

16. $(x^3 - 5x^2 - 7x + 25) \div (x - 5)$
$x^2 - 7$, R -10

17. $(x^4 - x^3 + x^2 - x + 1) \div (x - 1)$
$x^3 + x$, R 1

18. $(2x^4 + 7x^3 - 11x^2 + 21x + 5) \div (x + 5)$
$2x^3 - 3x^2 + 4x + 1$

19. $(x^4 - 5x^3 + 5x^2 + 7x - 12) \div (x - 4)$
$x^3 - x^2 + x + 11$, R 32

20. $(2x^4 + 23x^3 + 60x^2 - 125x - 500) \div (x + 4)$
$2x^3 + 15x^2 - 125$

Use synthetic division and the given factor to completely factor each polynomial function.

21. $y = x^3 + 3x^2 - 13x - 15; (x + 5)$
$y = (x + 1)(x - 3)(x + 5)$

22. $y = x^3 - 3x^2 - 10x + 24; (x - 2)$
$y = (x - 2)(x + 3)(x - 4)$

23. $y = x^3 + x^2 - 10x + 8; (x - 1)$
$y = (x - 2)(x + 4)(x - 1)$

24. $y = x^3 + 4x^2 - 9x - 36; (x + 3)$
$y = (x + 4)(x - 3)(x + 3)$

25. The expression $V(x) = x^3 - 13x + 12$ represents the volume of a rectangular safe in cubic feet. The length of the safe is $x + 4$. What linear expressions with integer coefficients could represent the other dimensions of the safe? Assume that the height is greater than the width.
height = $(x - 1)$ ft; width = $(x - 3)$ ft

Use synthetic division and the Remainder Theorem to find $P(a)$.

26. $P(x) = 3x^3 - 4x^2 - 5x + 1; a = 2$
-1

27. $P(x) = x^3 + 7x^2 + 12x - 3; a = -5$
-13

28. $P(x) = x^3 + 6x^2 + 10x + 3; a = -3$
0

29. $P(x) = 2x^4 - 9x^3 + 7x^2 - 5x + 11; a = 4$
39

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5-4 Practice (continued) *Form G*

Dividing Polynomials

Divide.

30. $(6x^3 + 2x^2 - 11x + 12) \div (3x + 4)$
$2x^2 - 2x - 1$, R 16

31. $(x^4 + 2x^3 + x - 3) \div (x - 1)$
$x^3 + 3x^2 + 3x + 4$, R 1

32. $(2x^4 + 3x^3 - 4x^2 + x + 1) \div (2x - 1)$
$x^3 + 2x^2 - x$, R 1

33. $(x^5 - 1) \div (x - 1)$
$x^4 + x^3 + x^2 + x + 1$

34. $(x^4 - 3x^2 - 10) \div (x - 2)$
$x^3 + 2x^2 + x + 2$, R -6

35. $(3x^3 - 2x^2 + 2x + 1) \div \left(x + \frac{1}{3}\right)$
$3x^2 - 3x + 3$

36. The volume in cubic inches of a box can be expressed as the product of its three dimensions: $V(x) = x^3 - 16x^2 + 79x - 120$. The length is $x - 8$. Find linear expressions with integer coefficients for the other dimensions. Assume that the width is greater than the height. **width: $x - 3$; height: $x - 5$**

37. Writing What are the divisor, quotient, and remainder represented by the synthetic division below?

-5	1	0	-19	30
		-5	25	-30
	1	-5	6	0

divisor $= x + 5$; quotient $= x^2 - 5x + 6$; remainder = 0

38. Reasoning What does it mean if $P(-4)$ for the polynomial function $P(x) = x^3 + 11x^2 + 34x + 24$ equals zero?
It means that $(x + 4)$ is a factor of the polynomial.

39. Error Analysis Using synthetic division, you say that the quotient of $4x^3 - 3x^2 + 15$ divided by $x - 1$ is $4x^2 - 7x + 7$ R 8. Your friend says that the quotient is $4x^2 + x + 1$ R 16. Who is correct? What mistake was made?
Your friend is correct. You forgot to change the sign of the divisor from negative to positive.

40. What is $P(-2)$ for $P(x) = 3x^3 - 6x^2 + 2x - 12$?
-64

41. The expression $x^3 + 16x^2 + 68x + 80$ represents the volume of a flower box in cubic inches. The expression $x + 4$ represents the depth of the box. Assume that the length is greater than the height and that linear expressions with integer coefficients represent both.

a. What are the other dimensions of the flower box? **length $= (x + 10)$ in.; height $= (x + 2)$ in.**

b. If $x = 3$, what are the dimensions of the flower box? **depth = 7 in.; length = 13 in.; height = 5 in.**

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5-4 Practice *Form K*

Dividing Polynomials

Divide using long division. Check your answers.

1. $(2x^2 + 7x - 5) \div (x + 1)$

To start, divide $\frac{2x^2}{x} = 2x$

Then, multiply $2x(x + 1) = 2x^2 + 2x$.

$$\begin{array}{r} 2x \phantom{{}+7x-5} \\ x+1 \overline{)\, 2x^2 + 7x - 5} \\ \underline{2x^2 + 2x} \phantom{{}-5} \end{array}$$

$2x + 5$, R -10

2. $(x^3 + x^2 - 14x - 27) \div (x + 3)$

$x^2 - 2x - 8$, R -3

3. $(2x^3 + 13x^2 + 16x + 5) \div (x + 5)$

$2x^2 + 3x + 1$

4. $(x^2 + 9x + 22) \div (x + 2)$

$x + 7$, R 8

5. $(6x^2 + 4x - 16) \div (2x - 2)$

$3x + 5$, R -6

6. $(8x^3 + 18x^2 + 7x - 3) \div (4x - 1)$

$2x^2 + 5x + 3$

7. $(12x^2 + 18x - 17) \div (6x - 3)$

$2x + 4$, R -5

Determine whether each binomial is a factor of $x^3 - 3x^2 - 4x$.

8. $x - 4$

yes

9. $x + 2$

no

10. $x - 3$

no

11. $x + 1$

yes

Determine whether each binomial is a factor of $x^3 - 9x^2 + 15x + 25$.

12. $x - 2$

no

13. $x + 1$

yes

14. $x - 5$

yes

15. $x - 3$

no

Name ______________________ Class ______________ Date ______________

5-4 Practice (continued) Form K

Dividing Polynomials

Divide using synthetic division.

16. $(x^3 - 7x^2 - 36) \div (x - 2)$

To start, write the coefficients of the polynomial. Use 2 for the divisor.

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 0 & -36 \\ & & 2 & -10 & -20 \\ \hline & 1 & -5 & -10 & -56 \end{array}$$

$x^2 - 5x - 10$, R -56

17. $(x^3 + x^2 - 14x - 27) \div (x + 3)$

$x^2 - 2x - 8$, R -3

18. $(x^3 - 6x^2 + 3x - 2) \div (x - 2)$

$x^2 - 4x - 5$, R -12

19. $(x^3 - 15) \div (x - 1)$

$x^2 + x + 1$, R -14

20. $(x^2 + 8) \div (x - 4)$

$x + 4$, R 24

21. $(3x^3 - 70x + 2) \div (x - 5)$

$3x^2 + 15x + 5$, R 27

22. $(2x^3 + x^2 - 8x + 4) \div (x + 2)$

$2x^2 - 3x - 2$, R 8

Use synthetic division and the given factor to completely factor each polynomial function.

23. $y = 2x^3 + 9x^2 + 13x + 6; (x + 1)$

$y = (x + 1)(x + 2)(2x + 3)$

24. $y = x^3 + 4x^2 - 7x - 10; (x - 2)$

$y = (x + 5)(x + 1)(x - 2)$

Use synthetic division and the Remainder Theorem to find $P(a)$.

25. $P(x) = 5x^3 - 12x^2 + 2x + 1, a = 3$

$P(3) = 34$

26. $P(x) = 2x^3 - 4x^2 + 3x - 6, a = -2$

$P(-2) = -44$

27. $P(x) = x^3 + 6x^2 - 2, a = 3$

$P(3) = 79$

28. $P(x) = 7x^3 + x^2 - 2x + 10, a = 1$

$P(1) = 16$

29. $P(x) = x^3 - 412, a = 8$

$P(8) = 100$

30. $P(x) = 2x^3 + x^2 - 3x - 3, a = -3$

$P(-3) = -39$

5-4 Standardized Test Prep

Dividing Polynomials

Gridded Response

Solve each exercise and enter your answer in the grid provided.

1. What is $P(-2)$ given that $P(x) = x^4 - 3x^2 + 5x + 10$?

2. What is the missing value in the following synthetic division?

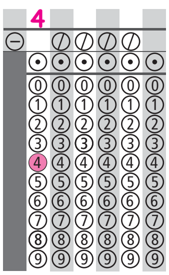
$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -5 & 4 & 12 \\ & & -4 & \boxed{} & -44 & 160 \\ \hline & 1 & -4 & 11 & -40 & 172 \end{array}$$

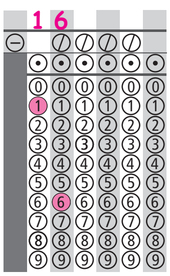
3. What is the remainder when $x^6 - 4x^4 + 4x^2 - 10$ is divided by $x + 3$?

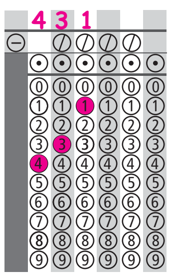
4. How many unique factors does $x^4 + 4x^3 - 3x^2 - 14x - 8$ have, including $(x + 4)$?

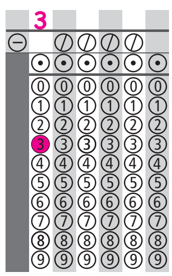
5. How many terms are there in the simplified form of $\dfrac{x^4 - 2x^3 - 23x^2 - 12x + 36}{x - 6}$?

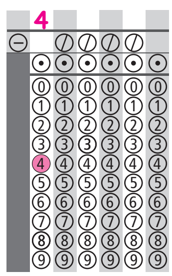
Answers

1.


2.


3.


4.


5.


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5-4 Enrichment

Dividing Polynomials

Synthetic division clearly simplifies the long division process for dividing by a linear expression $x - a$, but is there a way to use synthetic division when dividing by a linear expression of the form $ax - b$ where $a > 1$?

1. Use long division to divide $6x^3 - 11x^2 - 5x + 12$ by $2x - 3$.
$3x^2 - x - 4$

2. Use synthetic division to divide $6x^3 - 11x^2 - 5x + 12$ by $x - \frac{3}{2}$.
$6x^2 - 2x - 8$

3. How does the divisor in Exercise 2, $x - \frac{3}{2}$, compare to the divisor, $2x - 3$, in Exercise 1?
The divisor in Exercise 2 is $\frac{1}{2}$ the divisor in Exercise 1.

4. How does the quotient in Exercise 2 compare to the quotient in Exercise 1?
The quotient in Exercise 2 is twice the quotient in Exercise 1.

5. In order to use synthetic division to divide $6x^3 - 11x^2 - 5x + 12$ by $2x - 3$, you can divide by $x - \frac{3}{2}$. How will you adjust your quotient in order to have the correct answer?
Because the divisor was divided by two, the quotient will be twice as big, so divide the quotient by two in order to have the correct answer.

6. Describe how you can use synthetic division to divide $3x^3 - x^2 - 75x + 25$ by $3x - 1$. What is the quotient?
Divide the divisor by 3, use synthetic division to divide by $\frac{1}{3}$, and then divide the quotient by 3; $x^2 - 25$.

Divide using synthetic division.

7. $(4x^3 + 7x^2 - 62x + 15) \div (4x - 1)$
$x^2 + 2x - 15$

8. $(2x^3 + 17x^2 + 48x + 45) \div (2x + 5)$
$x^2 + 6x + 9$

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5-4 Reteaching

Dividing Polynomials

Problem

What is the quotient and remainder? Use polynomial long division to divide $2x^2 + 6x - 7$ by $x + 1$.

Step 1 To find the first term of the quotient, divide the highest-degree term of $\mathbf{2x^2} + 6x + 7$ by the highest-degree term of the divisor, $\boldsymbol{x} + 1$. Circle these terms before dividing.

$$\begin{array}{r} 2x \\ x + 1\overline{)2x^2 + 6x + 7} \end{array}$$

Step 2 Multiply $x + 1$ by the new term, $2x$, in the quotient. $2x(x + 1) = 2x^2 + 2x$. Align like terms.

$$\begin{array}{r} 2x \\ x + 1\overline{)2x^2 + 6x + 7} \\ \underline{2x^2 + 2x } \end{array}$$

Step 3 Subtract to get $4x$. Bring down the next term, 7.

$$\begin{array}{r} 2x \\ x + 1\overline{)2x^2 + 6x + 7} \\ \underline{2x^2 + 2x } \\ 4x + 7 \end{array}$$

Step 4 Divide the highest-degree term of $\mathbf{4x} + 7$ by the highest-degree term of $\boldsymbol{x} + 1$. Circle these terms before dividing.

Step 5 Repeat Steps 2 and 3. The *remainder* is 3 because its degree is less than the degree of $x + 1$.

$$\begin{array}{r} 2x + 4 \\ x + 1\overline{)2x^2 + 6x + 7} \\ \underline{2x^2 + 2x } \\ 4x + 7 \\ \underline{4x + 4} \\ 3 \end{array}$$

$2x^2 + 6x + 7$ divided by $x + 1$ is $2x + 4$, with a remainder of 3.
The quotient is $2x + 4$ with remainder 3.

Check the answer by multiplying $(x + 1)$ by $(2x + 4)$ and adding 3.
$(x + 1)(2x + 4) + 3 = 2x^2 + 6x + 7$

Exercises

Divide using polynomial long division.

1. $(3x^2 - 8x + 7) \div (x - 1)$
$3x - 5$, R 2

2. $(x^3 + 5x^2 - 3x - 4) \div (x + 6)$
$x^2 - x + 3$, R -22

3. $(x^2 + 3x - 8) \div (x - 5)$
$x + 8$, R 32

4. $(x^2 + 6x + 14) \div (x + 3)$
$x + 3$, R 5

5. $(x^3 - 7x^2 + 11x + 3) \div (x - 3)$
$x^2 - 4x - 1$

6. $(2x^3 - 3x^2 - x - 2) \div (x - 2)$
$2x^2 + x + 1$

7. $(2x^2 - 4x + 7) \div (x - 3)$
$2x + 2$, R 13

8. $(x^3 + 2x^2 - 20x + 4) \div (x + 7)$
$x^2 - 5x + 15$, R -101

9. $(x^2 - 5x + 2) \div (x - 1)$
$x - 4$, R -2

10. $(2x^3 + 3x^2 + x + 6) \div (x + 3)$
$2x^2 - 3x + 10$, R -24

5-4 Reteaching (continued)

Dividing Polynomials

Problem

Use synthetic division to divide $x^3 + 13x^2 + 46x + 48$ by $x + 3$. What is the quotient and remainder?

Step 1 Set up your polynomial division.
$(x^3 + 13x^2 + 46x + 48) \div (x + 3)$

Step 2 Reverse the sign of the constant, 3, in the divisor. Write the coefficients of the dividend: 1 13 46 48.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & & & \\ \hline & & & & \end{array}$$

Step 3 Bring the first coefficient, 1, down to the bottom line.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & & & \\ \hline & 1 & & & \end{array}$$

Step 4 Multiply the coefficient, 1, by the divisor, -3. Put this product, -3, underneath the second coefficient, 13, and add those two numbers: $13 + (-3) = 10$.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & -3 & & \\ \hline & 1 & 10 & & \end{array}$$

Step 5 Continue multiplying and adding through the last coefficient. The final sum is the remainder.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & -3 & -30 & -48 \\ \hline & 1 & 10 & 16 & 0 \end{array}$$

The quotient is $x^2 + 10x + 16$. Since the remainder is 0, $x + 3$ is a factor of $x^3 + 13x^2 + 46x + 48$.

Exercises

What is the quotient and remainder of the following polynomials?

11. $(x^3 - 2x + 8) \div (x + 2)$
$x^2 - 2x + 2$, R 4

12. $(12x^3 - 71x^2 + 57x - 10) \div (x - 5)$
$12x^2 - 11x + 2$, R 0

13. $(3x^4 + x^3 - 6x^2 - 9x + 12) \div (x + 1)$
$3x^3 - 2x^2 - 4x - 5$, R 17

14. $(2x^3 - 15x + 23) \div (x - 2)$
$2x^2 + 4x - 7$, R 9

15. $(x^3 + x + 10) \div (x + 2)$
$x^2 - 2x + 5$, R 0

16. $(x^4 - 12x^3 - 18x^2 + 10) \div (x + 4)$
$x^3 - 16x^2 + 46x - 184$, R 746

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5-5 Additional Vocabulary Support

Theorems About Roots of Polynomial Equations

Problem

What are the rational roots of $6x^3 - 17x^2 - 4x + 3 = 0$? Explain your work.

$6x^3 - 17x^2 - 4x + 3 = 0$	Write the original equation.
Constant term: $\pm 1, \pm 3$	Identify the factors of the constant term.
Leading coefficient: $\pm 1, \pm 2, \pm 3, \pm 6$	Identify the factors of the leading coefficient.
$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 3, \pm\frac{3}{2}$	Use the Rational Root Theorem to identify the possible rational roots.
$6(3)^3 - 17(3)^2 - 4(3) + 3 = 0$	Test the possible rational roots.
$\begin{array}{r\|rrrr} 3 & 6 & -17 & -4 & 3 \\ & & 18 & 3 & -3 \\ \hline & 6 & 1 & -1 & 0 \end{array}$	Factor the polynomial using synthetic division.
$P(x) = (x - 3)(6x^2 + x - 1)$	
$6x^2 + x - 1 = (3x - 1)(2x + 1)$	Find the other rational roots.
$3\left(\frac{1}{3}\right) - 1 = 0$	
$2\left(-\frac{1}{2}\right) + 1 = 0$	

The rational roots are 3, $\frac{1}{3}$, and $-\frac{1}{2}$.

Exercise

What are the rational roots of $4x^3 - 8x^2 + x + 3 = 0$? Explain your work.

$4x^3 - 8x^2 + x + 3 = 0$	**Write the original equation.**
Constant term: $\pm 1, \pm 3$	**Identify the factors of the constant term.**
Leading coefficient: $\pm 1, \pm 2, \pm 4$	**Identify the factors of the leading coefficient.**
$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}$	**Use the Rational Root Theorem to identify the possible rational roots.**
$4(1)^3 - 8(1)^2 + 1 + 3 = 0$	**Test the possible rational roots.**
$\begin{array}{r\|rrrr} 1 & 4 & -8 & 1 & 3 \\ & & 4 & -4 & -3 \\ \hline & 4 & -4 & -3 & 0 \end{array}$	**Factor the polynomial using synthetic division.**
$P(x) = (x - 1)(4x^2 - 4x - 3)$	
$4x^2 - 4x - 3 = (2x + 1)(2x - 3)$	**Find the other rational roots.**
$2\left(-\frac{1}{2}\right) + 1 = 0$	
$2\left(\frac{3}{2}\right) - 3 = 0$	

The rational roots are 1, $-\frac{1}{2}$, and $\frac{3}{2}$.

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5-5 Think About a Plan

Theorems About Roots of Polynomial Equations

Gardening A gardener is designing a new garden in the shape of a trapezoid. She wants the shorter base to be twice the height and the longer base to be 4 feet longer than the shorter base. If she has enough topsoil to create a 60 ft^2 garden, what dimensions should she use for the garden?

Understanding the Problem

1. What is the formula for the area of a trapezoid? $A = \frac{1}{2}h(b_1 + b_2)$

2. How can drawing a diagram help you solve the problem?

   Answers may vary. Sample: A diagram will help me write variable expressions for each part of the formula for the area of a trapezoid.

3. What is the problem asking you to determine?

   all of the dimensions of a trapezoid with area 60 ft^2

Planning the Solution

4. Define a variable. Let x = $\boxed{\text{height}}$.

5. What variable expression represents the shorter base? The longer base?

   $\boxed{2x}$ $\boxed{2x + 4}$

6. What expression represents the area of the trapezoid? What number is this equal to? Write the equation you obtain in standard form.

   $\frac{1}{2}x(4x + 4)$; 60; $2x^2 + 2x - 60 = 0$

Getting an Answer

7. Solve your equation. Are the solutions reasonable?

   -6, 5; Only the positive solution is reasonable for the height.

8. What are the dimensions of the garden?

   height = 5 ft, shorter base = 10 ft, longer base = 14 ft

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5-5 Practice

Form G

Theorems About Roots of Polynomial Equations

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual rational roots.

1. $x^3 + 5x^2 - 2x - 15 = 0$
$\pm1, \pm3, \pm5, \pm15$; none

2. $36x^3 + 144x^2 - x - 4 = 0$
$\pm1, \pm2, \pm4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{1}{12}, \pm\frac{1}{18}, \pm\frac{1}{36}$; $-4, -\frac{1}{6}, \frac{1}{6}$

3. $2x^3 + 5x^2 + 4x + 1 = 0$
$\pm1, \pm\frac{1}{2}$; $-1, -\frac{1}{2}$

4. $12x^4 + 14x^3 - 5x^2 - 14x - 4 = 0$
$\pm1, \pm2, \pm4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$; none

5. $5x^3 - 11x^2 + 7x - 1 = 0$
$\pm1, \pm\frac{1}{5}$; $\frac{1}{5}, 1$

6. $x^3 + 81x^2 - 49x - 49 = 0$
$\pm1, \pm7, \pm49$; none

A polynomial function $P(x)$ with rational coefficients has the given roots. Find two additional roots of $P(x) = 0$.

7. $2 + 3i$ and $\sqrt{7}$ **$2 - 3i, -\sqrt{7}$**

8. $3 - \sqrt{2}$ and $1 + \sqrt{3}$ **$3 + \sqrt{2}, 1 - \sqrt{3}$**

9. $-4i$ and $6 - i$ **$4i, 6 + i$**

10. $5 - \sqrt{6}$ and $-2 + \sqrt{10}$ **$5 + \sqrt{6}, -2 - \sqrt{10}$**

11. $\sqrt{5}$ and $-\sqrt{13}$ **$-\sqrt{5}$ and $\sqrt{13}$**

12. $1 - \sqrt{10}$ and $2 + \sqrt{2}$ **$1 + \sqrt{10}$ and $2 - \sqrt{2}$**

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

13. 4 and 6 **$P(x) = x^2 - 10x + 24$**

14. -5 and -1 **$P(x) = x^2 + 6x + 5$**

15. $3i$ and $\sqrt{6}$ **$P(x) = x^4 + 3x^2 - 54$**

16. $2 + i$ and $1 - \sqrt{5}$
$P(x) = x^4 - 6x^3 + 9x^2 + 6x - 20$

17. -5 and $3i$ **$P(x) = x^3 + 5x^2 + 9x + 45$**

18. i and $5i$ **$P(x) = x^4 + 26x^2 + 25$**

What does Descartes' Rule of Signs say about the number of positive real roots and negative real roots for each polynomial function?

19. $P(x) = 3x^3 + x^2 - 8x - 12$ **1 positive real root; 2 or 0 negative real roots**

20. $P(x) = 2x^4 - x^3 - 3x + 7$ **2 or 0 positive real roots; 0 negative real roots**

21. $P(x) = 4x^5 - x^4 - x^3 + 6x^2 - 5$ **3 or 1 positive real roots; 2 or 0 negative real roots**

22. $P(x) = x^3 + 4x^2 + x - 6$ **1 positive real root; 2 or 0 negative real roots**

5-5 Practice (continued) *Form G*

Theorems About Roots of Polynomial Equations

Find all rational roots for $P(x) = 0$.

23. $P(x) = x^3 - 5x^2 + 2x + 8$ **4, 2, −1**

24. $P(x) = x^3 + x^2 - 17x + 15$ **3, 1, −5**

25. $P(x) = 2x^3 + 13x^2 + 17x - 12$ **$-4, -3, \frac{1}{2}$**

26. $P(x) = x^3 - x^2 - 34x - 56$ **7, −2, −4**

27. $P(x) = x^3 - 18x + 27$ **3**

28. $P(x) = x^4 - 5x^2 + 4$ **−2, −1, 1, 2**

29. $P(x) = x^3 - 6x^2 + 13x - 10$ **2**

30. $P(x) = x^3 - 5x^2 + 4x + 10$ **−1**

31. $P(x) = x^3 - 5x^2 + 17x - 13$ **1**

32. $P(x) = x^3 + x + 10$ **−2**

33. $P(x) = x^3 - 5x^2 - x + 5$ **1, −1, 5**

34. $P(x) = x^3 - 12x + 16$ **−4, 2**

35. $P(x) = x^3 - 2x^2 - 5x + 6$ **−2, 1, 3**

36. $P(x) = x^3 - 8x^2 - 200$ **10**

37. $P(x) = x^3 + x^2 - 5x + 3$ **1, −3**

38. $P(x) = 4x^3 - 12x^2 - x + 3$ **$3, \frac{1}{2}, -\frac{1}{2}$**

39. $P(x) = x^3 + x^2 - 7x + 2$ **2**

40. $P(x) = 12x^3 + 31x^2 - 17x - 6$ **$-3, \frac{2}{3}, -\frac{1}{4}$**

Write a polynomial function $P(x)$ with rational coefficients so that $P(x) = 0$ has the given roots.

41. $\sqrt{3}, 2, -i$
$P(x) = x^5 - 2x^4 - 2x^3 + 4x^2 - 3x + 6$

42. $5, 2i$
$P(x) = x^3 - 5x^2 + 4x - 20$

43. $-1, 3 + i$
$P(x) = x^3 - 5x^2 + 4x + 10$

44. $-\sqrt{7}, i$
$P(x) = x^4 - 6x^2 - 7$

45. $-4, 4i$
$P(x) = x^3 + 4x^2 + 16x + 64$

46. $6, 3 - 2i$
$P(x) = x^3 - 12x^2 + 49x - 78$

47. **Error Analysis** A student claims that $2i$ is the only imaginary root of a polynomial equation that has real coefficients. Explain the student's mistake.
The student forgot the conjugate imaginary root $-2i$.

48. You are building a rectangular sandbox for a children's playground. The width of the sandbox is 4 times its height. The length of the sandbox is 8 ft more than 2 times its height. You have 40 ft^3 of sand available to fill this sandbox. What are the dimensions of the sandbox? **height = 1 ft, width = 4 ft, length =10 ft**

49. **Writing** According to the Rational Root Theorem, what is the relationship between the polynomial equation $2x^4 - x^3 - 7x^2 + 5x + 3 = 0$ and rational roots of the form $\frac{p}{q}$, where $\frac{p}{q}$ is in simplest form?
p must be a factor of 3 and q must be a factor of 2.

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5-5 Practice

Form K

Theorems About Roots of Polynomial Equations

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual rational roots.

1. $x^3 - 5x^2 + 17x - 13$

To start, list the constant term's factors and the leading coefficient's factors.

constant term factors: $\pm 1, \pm 13$

leading coefficient factors: ± 1

$\pm 1, \pm 13; 1$

2. $2x^3 - 5x^2 + x - 7$

$\pm 1, \pm \frac{1}{2}, \pm \frac{7}{2}, \pm 7$; none

3. $x^3 - 4x^2 - 15x + 18$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18; -3, 1, 6$

4. $x^3 - 8x^2 - 2$

$\pm 1, \pm 2$; none

5. $x^3 - x^2 + 6x - 6$

$\pm 1, \pm 2, \pm 3, \pm 6; 1$

6. $4x^3 + 12x^2 + x + 3$

$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}; -3$

7. $x^3 - 3x^2 - 16x - 12$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12; -1, -2, 6$

8. $x^3 + 8x^2 - x - 8$

$\pm 1, \pm 2, \pm 4, \pm 8; -8, -1, 1$

9. $x^3 - 3x^2 - 24x - 28$

$\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28; -2, 7$

Find all rational roots for $P(x) = 0$.

10. $P(x) = x^3 + 5x^2 + 2x - 8$

$-4, -2, 1$

11. $P(x) = x^4 - 4x^3 - 13x^2 + 4x + 12$

$-2, -1, 1, 6$

12. $P(x) = x^3 + 14x^2 + 53x + 40$

$-8, -5, -1$

13. $P(x) = x^3 + 3x^2 - 4x - 12$

$-3, -2, 2$

14. $P(x) = x^3 + 5x^2 - 9x - 45$

$-5, -3, 3$

15. $P(x) = x^3 + 9x^2 - x - 9$

$-9, -1, 1$

16. $P(x) = x^3 - 7x^2 - x + 7$

$-1, 1, 7$

17. $P(x) = x^3 - 7x^2 + 14x - 8$

$1, 2, 4$

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5-5 Practice (continued) Form K

Theorems About Roots of Polynomial Equations

A polynomial function $P(x)$ with rational coefficients has the given roots. Find two additional roots of $P(x) = 0$.

18. $1 + 4i$ and $\sqrt{3}$
$1 - 4i$ and $-\sqrt{3}$

19. $3 - \sqrt{2}$ and $1 + \sqrt{3}$
$3 + \sqrt{2}$ and $1 - \sqrt{3}$

20. $-8i$ and $7 - i$
$8i$ and $7 + i$

21. $6 - \sqrt{7}$ and $-3 + \sqrt{10}$
$6 + \sqrt{7}$ and $-3 - \sqrt{10}$

22. $\sqrt{2}$ and $-\sqrt{13}$
$-\sqrt{2}$ and $\sqrt{13}$

23. $1 - \sqrt{3}$ and $1 + \sqrt{2}$
$1 + \sqrt{3}$ and $1 - \sqrt{2}$

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

24. $3i$

To start, use the Conjugate Root Theorem to identify a second root.

Since $3i$ is a root, $-3i$ is also a root.

$P(x) = x^2 + 9$

25. -2 and -8
$P(x) = x^2 + 10x + 16$

26. 4 and 1
$P(x) = x^2 - 5x + 4$

27. $2i$ and $\sqrt{2}$
$P(x) = x^4 + 2x^2 - 8$

28. $3 + i$ and $1 - \sqrt{3}$
$P(x) = x^4 - 8x^3 + 20x^2 - 8x - 20$

29. -4 and $5i$
$P(x) = x^3 + 4x^2 + 25x + 100$

30. $2i$ and i
$P(x) = x^4 + 5x^2 + 4$

What does Descartes' Rule of Signs say about the number of positive real roots and negative real roots for each polynomial function?

31. $P(x) = x^3 - x^2 - 8x + 12$

To start, count and identify the number of sign changes in $P(x)$.

There are 2 sign changes in $P(x)$. So there are 0 or 2 positive real roots.

2 or 0 positive real roots; 1 negative real roots

32. $P(x) = 2x^3 + 2x^2 - 5x - 2$
1 positive real root;
2 or 0 negative real roots

33. $P(x) = x^4 - 3x^3 - x + 5$
2 or 0 positive real roots;
0 negative real roots

5-5 Standardized Test Prep

Theorems About Roots of Polynomial Equations

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. A fourth-degree polynomial with integer coefficients has roots at 1 and $3 + \sqrt{5}$. Which number *cannot* also be a root of this polynomial? **D**

(A) -1 (B) -3 (C) $3 - \sqrt{5}$ (D) $3 + \sqrt{2}$

2. A quartic polynomial $P(x)$ has rational coefficients. If $\sqrt{7}$ and $6 + i$ are roots of $P(x) = 0$, what is one additional root? **G**

(F) 7 (G) $-\sqrt{7}$ (H) $i - 6$ (I) $6i$

3. What is a quartic polynomial function with rational coefficients that has roots i and $2i$? **C**

(A) $x^4 - 5x^2 - 4$ (B) $x^4 - 5x^2 + 4$ (C) $x^4 + 5x^2 + 4$ (D) $x^4 + 5x^2 - 4$

4. What does Descartes' Rule of Signs tell you about the real roots of $6x^4 + 29x^3 + 40x^2 + 7x - 12$? **F**

(F) 1 positive real root and 1 or 3 negative real roots

(G) 0 positive real roots and 1 negative real root

(H) 1 or 3 positive real roots and 1 negative real root

(I) 0 or 1 positive real roots and 3 negative real roots

5. What is a rational root of $x^3 + 3x^2 - 6x - 8 = 0$? **B**

(A) 1 (B) -1 (C) 8 (D) -8

Extended Response

6. A third-degree polynomial with rational coefficients has roots -4 and $-4i$. If the leading coefficient of the polynomial is $\frac{3}{2}$, what is the polynomial? Show your work.

[4] 3rd root $= 4i$; $P(x) = \frac{3}{2}(x + 4)(x + 4i)(x - 4i) = \frac{3}{2}(x + 4)(x^2 + 16) = \frac{3}{2}x^3 + 6x^2 + 24x + 96$

[3] correct process, but with one computational error

[2] correct process, with multiple computational errors

[1] correct answer without work shown

[0] incorrect answers and no work shown OR no answers given

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5-5 Enrichment

Theorems About Roots of Polynomial Equations

Lower and Upper Bounds for Real Roots of Polynomial Equations

At times, the list of possible rational roots for a polynomial equation is rather lengthy. However, you can use patterns to shorten the list. One pattern involves finding lower and upper bounds for real roots. If a number is an upper bound, then there are no real roots for the equation greater than that number. If a number is a lower bound, then there are no real roots for the equation lower than that number.

Consider the polynomial equation $x^3 + 3x^2 - 34x - 42 = 0$

1. List all the possible rational zeros for the equation. **±1, ±2, ±3, ±6, ±7, ±14, ±21, ±42**

When testing the possible positive rational roots, if the last row of numbers in the synthetic division table are all positive, then the number is an upper bound.

2. Use synthetic division to see whether 6 is a root of the given polynomial.

6	1	3	−34	−42
		6	54	120
	1	9	20	78

Notice the last row contains all positive numbers. Therefore, 6 is an upper bound. We can rule out any other numbers in our possible rational zeros list that are greater than 6.

3. Which numbers can be eliminated from our list of possible rational roots? **7, 14, 21, 42**

When testing the possible negative rational roots, if the last row of numbers in the synthetic division table are of alternating signs, then the number is a lower bound.

4. Use synthetic division to see whether -14 is a root of the given polynomial.

−14	1	3	−34	−42
		−14	154	−1680
	1	−11	120	−1722

Notice the last row contains numbers with alternating signs. Therefore, -14 is a lower bound. We can rule out any other numbers in our possible rational roots list that are less than -14.

5. Which numbers can be eliminated from our list of possible rational roots? **−21, −42**

6. What numbers in our original list from Exercise 1 are still remaining? **±1, ±2, ±3, −6, −7**

7. Find all roots of the given polynomial. **$-7, 2 \pm \sqrt{10}$**

8. List all possible rational roots for the polynomial equation $x^4 + x^3 + 30x^2 + 36x - 216 = 0$. Then use synthetic division to find all zeros. Note any upper or lower bounds. **±1, ±2, ±3, ±4, ±6, ±8, ±9, ±12, ±18, ±24, ±27, ±36, ±54, ±72, ±108, ±216; $-3, 2, -6i, 6i$; upper: 3, lower: −4**

9. List all possible rational roots for the polynomial equation $x^4 - 3x^3 + 8x^2 - 36x - 48 = 0$. Then use synthetic division to find all zeros. Note any upper or lower bounds.
±1, ±2, ±3, ±4, ±6, ±8, ±12, ±16, ±24, ±48; $-1, 4, -2i\sqrt{3}, 2i\sqrt{3}$; upper: 6, lower: −2

10. List all possible rational roots for the polynomial equation $x^4 + 4x^3 - 29x^2 + 64x - 720 = 0$. Then use synthetic division to find all zeros. Note any upper or lower bounds.
±1, ±2, ±3, ±4, ±5, ±6, ±8, ±9, ±10, ±12, ±15, ±16, ±18, ±20, ±24, ±30, ±36, ±40, ±45, ±48, ±60, ±72, ±80, ±90, ±120, ±144, ±180, ±240, ±360, ±720; $-9, 5, -4i, 4i$; upper: 6, lower: −10

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5-5 Reteaching

Theorems About Roots of Polynomials Equations

Problem

What are the rational roots of $6x^4 + 29x^3 + 40x^2 + 7x - 12 = 0$?

Step 1 Determine the factors of the constant term and the factors of the leading coefficient.

constant term: 12 factors: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

leading coefficient: 6 factors: $\pm1, \pm2, \pm3, \pm6$

Step 2 Find all the possible roots by dividing the factors of the constant term by the factors of the leading coefficient. $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{6}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Step 3 Substitute each possible root into the polynomial until you find one that causes the polynomial to equal zero. This is one rational root.

Test $-\frac{3}{2}$: $6\left(-\frac{3}{2}\right)^4 + 29\left(-\frac{3}{2}\right)^3 + 40\left(-\frac{3}{2}\right)^2 + 7\left(-\frac{3}{2}\right) - 12 = 0$ $-\frac{3}{2}$ is a rational root.

Step 4 Factor the polynomial by synthetic division using the first rational root as the divisor.

$$\begin{array}{r|rrrrr} -\frac{3}{2} & 6 & 29 & 40 & 7 & -12 \\ & & -9 & -30 & -15 & 12 \\ \hline & 6 & 20 & 10 & -8 & 0 \end{array}$$

Step 5 If the dividend is a second-degree polynomial, factor to find any additional rational roots. If the dividend does not factor, there are no additional rational roots. If the dividend is greater than a second-degree polynomial, repeat Steps 1–4 until the dividend is a second-degree polynomial.

$$\begin{array}{r|rrrr} -\frac{4}{3} & 6 & 20 & 10 & -8 \\ & & -8 & -16 & 8 \\ \hline & 6 & 12 & -6 & 0 \end{array}$$

$6x^2 + 12x - 6 = 0$ does not factor. The rational roots of $6x^4 + 29x^3 + 40x^2 + 7x - 12$ are $-\frac{3}{2}$ and $-\frac{4}{3}$.

Exercises

Find all rational roots for $P(x) = 0$.

1. $P(x) = x^3 - x^2 - 8x + 12$ **−3, 2**

2. $P(x) = x^4 - 49x^2$ **0, 7, −7**

3. $P(x) = 2x^3 - 7x^2 - 21x + 54$ **−3, 2, $\frac{9}{2}$**

4. $P(x) = x^4 - 2x^3 - 3$ **−1**

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5-5 Reteaching (continued)

Theorems About Roots of Polynomials Equations

Problem

What is a third-degree polynomial function $y = P(x)$ with rational coefficients so that $P(x) = 0$ has roots -4 and $2 \pm 3i$?

Roots: $-4, 2 - 3i, 2 + 3i$	Because $2 - 3i$ is a root, its complex conjugate $2 + 3i$ is also a root.
$(x + 4)[x - (2 - 3i)][x - (2 + 3i)]$	Write the factored form of the polynomial.
$(x + 4)[x^2 - x(2 + 3i) - x(2 - 3i) + (2 - 3i)(2 + 3i)]$	Multiply the factors.
$(x + 4)[x^2 - 2x - 3ix - 2x + 3ix + 4 + 6i - 6i - 9i^2]$	Multiply.
$(x + 4)[x^2 - 4x + 4 - 9i^2]$	Simplify.
$(x + 4)(x^2 - 4x + 13)$	Combine like terms.
$x^3 + 4x^2 - 4x^2 - 16x + 13x + 52$	Multiply.
$x^3 - 3x + 52$	Combine like terms.

A third-degree polynomial function with rational coefficients so that $P(x) = 0$ has roots -4 and $2 \pm 3i$ is $P(x) = x^3 - 3x + 52$.

Exercises

Write a third-degree polynomial function $y = P(x)$ with rational coefficients so that $P(x) = 0$ has the given roots.

5. $1, 2 - i$ $P(x) = x^3 - 5x^2 + 9x - 5$

6. $5 + 2i, -2$ $P(x) = x^3 - 8x^2 + 9x + 58$

7. $3, 6 + i$ $P(x) = x^3 - 15x^2 + 73x - 111$

8. $-4, \sqrt{2}$ $P(x) = x^3 + 4x^2 - 2x - 8$

9. $2 - \sqrt{3}, -1$ $P(x) = x^3 - 3x^2 - 3x + 1$

10. $0, 3 - \sqrt{3}$ $P(x) = x^3 - 6x^2 + 6x$

11. $3i, 7$ $P(x) = x^3 - 7x^2 + 9x - 63$

12. $2 + \sqrt{5}, 3$ $P(x) = x^3 - 7x^2 + 11x + 3$

13. $-3, i$ $P(x) = x^3 + 3x^2 + x + 3$

14. $1 - i, 8$ $P(x) = x^3 - 10x^2 + 18x - 16$

15. $1, 5i$ $P(x) = x^3 - x^2 + 25x - 25$

16. $2, 4 + i$ $P(x) = x^3 - 10x^2 + 33x - 34$

17. $3, -4i$ $P(x) = x^3 - 3x^2 + 16x - 48$

18. $0, 2 - i$ $P(x) = x^3 - 4x^2 + 5x$

19. $-7, 1 - \sqrt{2}$ $P(x) = x^3 + 5x^2 - 15x - 7$

20. $-4, -\sqrt{7}$ $P(x) = x^3 + 4x^2 - 7x - 28$

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5-6 Additional Vocabulary Support

The Fundamental Theorem of Algebra

Choose the word from the list that best completes each sentence.

complex roots	degree	Fundamental Theorem of Algebra
Quadratic Formula	synthetic division	

1. The ___degree___ of a polynomial is the greatest degree among its monomial terms.

2. ___Synthetic division___ can be used to factor a polynomial.

3. The degree of a polynomial tells you how many ___complex roots___ the equation has.

4. The ___Fundamental Theorem of Algebra___ states that the number of complex roots of a polynomial equation is equal to the degree of the polynomial.

5. The ___Quadratic Formula___ can be used to find the complex roots of a quadratic equation.

Use your knowledge of the Fundamental Theorem of Algebra to answer the following question.

6. Which of the following statements does not correctly state the Fundamental Theorem of Algebra? B

   (A) Every polynomial equation of degree $n \geq 1$ with complex coefficients has exactly n complex roots, including multiple roots.

   (B) Every polynomial of degree $n \geq 1$ with complex coefficients has exactly $n + 1$ complex roots.

   (C) Every polynomial function of degree $n \geq 1$ with complex coefficients has at least one complex zero.

   (D) Every polynomial of degree $n \geq 1$ with complex coefficients has no less than one complex root.

Use your knowledge of the Fundamental Theorem of Algebra to identify the number of complex roots for each polynomial equation.

7. $x^4 + 2x^3 - x^2 + 2 = 0$ ___4 complex roots___

8. $x^5 - 3x^4 + 2x^2 + 5x - 1 = 0$ ___5 complex roots___

9. $x + 5 = 0$ ___1 complex root___

10. $x^2 - 4x - 2 = 0$ ___2 complex roots___

5-6 Think About a Plan

The Fundamental Theorem of Algebra

Bridges A twist in a river can be modeled by the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - x$, $-3 \leq x \leq 2$. A city wants to build a road that goes directly along the x-axis. How many bridges would it have to build?

Know

1. The function has exactly $\boxed{3}$ complex roots.

2. **Check students' work.**

3. **Check students' work.**

Need

4. To solve the problem I need to:

find the number of real roots of the function on the interval $-3 \leq x \leq 2$.

Plan

5. Graph the function on a graphing calculator. What viewing window should you use?

Answers may vary. Sample: Xmin = −5, Xmax = 5, Ymin = −5, Ymax = 5

6. What does the graph tell you?

The function has 3 real roots on the interval $-3 \leq x \leq 2$.

7. How many bridges would the city have to build? $\boxed{3}$
Explain.

Answers may vary. Sample: Because the function has 3 real roots, it crosses the x-axis 3 times. So, the road will cross the river 3 times.

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5-6 Practice

Form G

The Fundamental Theorem of Algebra

Without using a calculator, find all the complex roots of each equation.

1. $x^5 - 3x^4 - 8x^3 - 8x^2 - 9x - 5 = 0$
5, −1, −1, *i*, −*i*

2. $x^3 - 2x^2 + 4x - 8 = 0$
−2, 2*i*, −2*i*

3. $x^3 + x^2 - x + 2 = 0$
$-2, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$

4. $x^4 - 2x^3 - x^2 - 4x - 6 = 0$
$3, -1, i\sqrt{2}, -i\sqrt{2}$

5. $x^4 + 3x^3 - 21x^2 - 48x + 80 = 0$
$4, -4, \frac{-3 + \sqrt{29}}{2}, \frac{-3 - \sqrt{29}}{2}$

6. $x^5 - 3x^4 + x^3 + x^2 + 4 = 0$
2, 2, −1, *i*, −*i*

Find all the zeros of each function.

7. $y = 5x^3 - 5x$
−1, 0, 1

8. $f(x) = x^3 - 16x$
−4, 0, 4

9. $g(x) = 12x^3 - 2x^2 - 2x$
$-\frac{1}{3}, 0, \frac{1}{2}$

10. $y = 6x^3 + x^2 - x$
$-\frac{1}{2}, 0, \frac{1}{3}$

11. $f(x) = 5x^3 + 6x^2 + x$
$-1, -\frac{1}{5}, 0$

12. $y = -4x^3 + 100x$
−5, 0, 5

For each equation, state the number of complex roots, the possible number of real roots, and the possible rational roots.

13. $2x^2 + 5x + 3 = 0$
2; 2 or 0; $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}$

14. $3x^2 + 11x - 10 = 0$
2; 2 or 0; $\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

15. $2x^4 - 18x^2 + 5 = 0$
4; $4, 2$ or 0; $\pm1, \pm5, \pm\frac{1}{2}, \pm\frac{5}{2}$

16. $4x^3 - 12x + 9 = 0$
3; 3 or 1; $\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

17. $6x^5 - 28x + 15 = 0$ 5; $5, 3,$ or 1; $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

18. $x^3 - x^2 - 2x + 7 = 0$
3; $3,$ or 1; $\pm1, \pm7$

19. $x^3 - 6x^2 - 7x - 12 = 0$
3; 3 *or* 1; $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

20. $2x^4 + x^2 - x + 6 = 0$
4; $4, 2,$ or 0; $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

21. $4x^5 - 5x^4 + x^3 - 2x^2 + 2x - 6 = 0$
5; $5, 3,$ or 1; $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

22. $7x^6 + 3x^4 - 9x^2 + 18 = 0$ 6; $6, 4, 2$ or 0; $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm\frac{1}{7}, \pm\frac{2}{7}, \pm\frac{3}{7}, \pm\frac{6}{7}, \pm\frac{9}{7}, \pm\frac{18}{7}$

23. $5 + x + x^2 + x^3 + x^4 + x^5 = 0$
5; $5, 3,$ or 1; $\pm1, \pm5$

24. $6 - x + 2x^3 - x^3 + x^4 - 8x^5 = 0$ 5; $5, 3,$ or 1; $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$

Find the number of complex roots for each equation.

25. $x^8 - 5x^6 + x^4 + 2x - 16 = 0$ **8**

26. $x^{10} - 100 = 0$ **10**

27. $2x^4 + x^3 - 3x^2 + 4x - 2 = 0$ **4**

28. $-4x^3 + x^2 - 3x + 10 = 0$ **3**

29. $x^6 + 2x^5 + 3x^4 + 4x^3 + 5x^2 + 6x + 10 = 0$ **6**

30. $-3x^5 + 4x^4 + 5x^2 - 15 = 0$ **5**

5-6 Practice (continued) — Form G

The Fundamental Theorem of Algebra

Find all the zeros of each function.

31. $f(x) = x^3 - 9x^2 + 27x - 27$
3

32. $y = 2x^3 - 8x^2 + 18x - 72$
$4, -3i, 3i$

33. $y = x^3 - 10x - 12$
$-2, 1 \pm \sqrt{7}$

34. $y = x^3 - 4x^2 + 8$
$2, 1 \pm \sqrt{5}$

35. $f(x) = 2x^3 + x - 3$
$1, \frac{-1 \pm i\sqrt{5}}{2}$

36. $y = x^3 - 2x^2 - 11x + 12$
$-3, 1, 4$

37. $g(x) = x^3 + 4x^2 + 7x + 28$
$-4, -i\sqrt{7}, i\sqrt{7}$

38. $f(x) = x^3 + 3x^2 + 6x + 4$
$-1, -1 \pm i\sqrt{3}$

39. $g(x) = x^4 - 5x^2 - 36$
$-3, 3, -2i, 2i$

40. $y = x^4 - 7x^2 + 12$
$-2, 2, -\sqrt{3}, \sqrt{3}$

41. $y = 9x^4 + 5x^2 - 4$
$-\frac{2}{3}, \frac{2}{3}, -i, i$

42. $y = 4x^4 - 11x^2 - 3$
$-\sqrt{3}, \sqrt{3}, -\frac{1}{2}i, \frac{1}{2}i$

43. Error Analysis Your friend says that the equation $4x^7 - 3x^3 + 4x^2 - x + 2 = 0$ has 5 complex roots. You say that the equation has 7 complex roots. Who is correct? What mistake was made?
You are correct. Your friend may have counted the number of terms in the equation instead of using the Fundamental Theorem of Algebra.

44. A section of roller coaster can be modeled by the function $f(x) = x^5 - 5x^4 - 31x^3 + 113x^2 + 282x - 360$. A walkway bridge will be placed at one of the zeros. What are the possible locations for the walkway bridge? **$-4, -3, 1, 5, 6$**

45. Writing Using the Fundamental Theorem of Algebra, explain how $x^3 = 0$ has 3 roots and 3 linear factors. **The Fundamental Theorem of Algebra says that the degree of the function is equal to the number of zeros. The degree of $x^3 = 0$ is 3. x^3 can be written as $x \cdot x \cdot x = 0$ or $(x - 0)(x - 0)(x - 0) = 0$. This shows that there are 3 linear factors and 3 zeros, all equal to 0.**

46. How many complex roots does the equation $x^4 = 256$ have? What are they?
4 complex roots; $4, -4, 4i, -4i$

47. Reasoning Can a fifth-degree polynomial with rational coefficients have 4 real roots and 1 irrational root? Explain why or why not.
No; a polynomial with rational coefficients cannot have only 1 irrational root, it must have both conjugates. So, a fifth-degree polynomial can have 0, 2 or 4 irrational roots and 5, 3 or 1 real roots.

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5-6 Practice

Form K

The Fundamental Theorem of Algebra

Without using a calculator, find all the roots of each equation.

1. $x^3 - 5x^2 + x - 5 = 0$

To start, identify the possible rational roots.

The possible rational roots are $\pm 1, \pm 5$

$-i, i, 5$

2. $x^5 + 3x^4 - 8x^3 - 24x^2 - 9x - 27 = 0$

$-3, 3, i, -i$

3. $x^3 + 4x^2 + 9x + 36 = 0$

$-4, 3i, -3i$

4. $x^3 + x^2 - 2x - 2 = 0$

$-1, -\sqrt{2}, \sqrt{2}$

5. $x^4 + 15x^2 - 16 = 0$

$1, -1, 4i, -4i$

6. $x^4 - 8x^3 + 19x^2 - 32x + 60 = 0$

$3, 5, 2i, -2i$

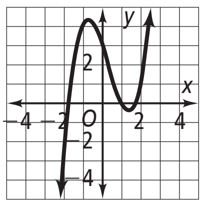
7. $x^3 + 5x^2 - 3x - 15 = 0$

$-5, -\sqrt{3}, \sqrt{3}$

Find all the zeros of each function.

8. $y = x^3 - x^2 - 3x + 3$

To start, use a graphing calculator to find the possible rational roots.



$-\sqrt{3}, 1, \sqrt{3}$

9. $y = x^4 - 4x^3 + 7x^2 - 16x + 12$

$1, 3, 2i, -2i$

10. $f(x) = x^3 + x^2 + 16x + 16$

$-1, 4i, -4i$

11. $g(x) = x^3 - 4x^2 + 4x - 3$

$3, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$

12. $y = x^3 + 6x^2 - 5x - 30$

$-6, \sqrt{5}, -\sqrt{5}$

13. $f(x) = x^4 - 2x^3 + 2x^2 - 2x + 1$

$1, i, -i$

14. $y = x^4 + 2x^3 - 5x^2 - 4x + 6$

$1, -3, \sqrt{2} -\sqrt{2}$

5-6 Practice (continued) *Form K*

The Fundamental Theorem of Algebra

For each equation, state the number of complex roots, the possible number of positive real roots, and the possible rational roots.

15. $x^2 + 8x - 5 = 0$
2; 1; ±1, ±5

16. $2x^3 - 18x + 4 = 0$
3; 2 or 0; $\pm\frac{1}{2}$, ±1, ±2, ±4

17. $x^4 + 8x^2 + 2 = 0$
4; 0; ±1, ±2

18. $x^6 - 8x^4 + 2x^2 - 10 = 0$
6; 3 or 1; ±1, ±2, ±5, ±10

19. $x^3 - 2x + 6 = 0$
3; 2 or 0; ±1, ±2, ±3, ±6

20. $8x + x^2 - 12 = 0$
2; 1; ±1, ±2, ±3, ±4, ±6, ±12

Find the number of complex roots for each equation.

21. $5x^6 + 3x^4 + x - 10 = 0$
6

22. $-4x^3 + 2x^2 - x + 5 = 0$
3

23. $2x^5 + 2x^3 - x^2 + 12x - 8 = 0$
5

24. $-x^3 + 7x^2 - 12x + 9 = 0$
3

25. $3x^8 + 4x^6 + 5x^2 - x + 15 = 0$
8

26. $12x^5 + 3x^4 + 2x^2 - 12 = 0$
5

27. $-5x^3 + 2x^3 + 2x - 32 = 0$
3

28. $x^{10} - 25 = 0$
10

29. Error Analysis Your friend says that the function $3x^4 - 2x^3 - x + 12 = 0$ has 3 complex roots. You say that the function has 4 complex roots. Who is correct? What mistake was made?
You are correct; your friend used the coefficient of the leading term in the function instead of using the Fundamental Theorem of Algebra.

30. A section of a bridge can be modeled by the function $f(x) = x^4 - 5x^3 - 10x^2 + 20x + 24$. Support beams for this bridge will be placed at one of the zeros. What are the possible locations for the support beams?
−2, −1, 2, 6

31. How many complex roots does the equation $x^4 = 81$ have? What are they?
4 complex roots; 3, −3, 3*i*, −3*i*

5-6 Standardized Test Prep

The Fundamental Theorem of Algebra

Multiple Choice

For Exercises 1–6, choose the correct letter.

1. Which number is a zero of $f(x) = x^3 + 6x^2 + 9x$ with multiplicity 1? **B**

(A) -3 (B) 0 (C) 1 (D) 3

2. One root of the equation $x^3 + x^2 - 2 = 0$ is 1. What are the other two roots? **F**

(F) $-1 \pm i$ (G) $1 \pm 2i$ (H) $\pm 1 + 2i$ (I) $\pm 1 - i$

3. A polynomial with real coefficients has 3, $2i$, and $-i$ as three of its zeros. What is the least possible degree of the polynomial? **C**

(A) 3 (B) 4 (C) 5 (D) 6

4. How many times does the graph of $x^3 + 27$ cross the x-axis? **G**

(F) 0 (G) 1 (H) 2 (I) 4

5. Which of the following is the polynomial with zeros at 1, $-\frac{3}{2}$, $2i$, and $-2i$? **A**

(A) $2x^4 + x^3 + 5x^2 + 4x - 12$ (C) $2x^4 + x^3 - 11x^2 - 4x + 12$

(B) $2x^4 - x^3 + 5x^2 - 4x - 12$ (D) $2x^4 - x^3 - 11x^2 + 4x + 12$

6. A polynomial with real coefficients has roots of 6, -2, $-4i$, and $\sqrt{5}$. Which of the following *must* be another root of this polynomial? **I**

(F) -6 (G) $-\sqrt{5}$ (H) 2 (I) $4i$

Short Response

7. One root of the equation $x^4 - 4x^3 - 6x^2 + 4x + 5 = 0$ is -1. How many complex roots does this equation have? What are all the roots? Show your work. **There are 4 complex roots.**

[2] Use known zero as divisor and synthetic division to begin factoring. Then use Rational Root Theorem to determine that ±1 and ±5 are possible rational roots. Try each possible rational root until you find a root. Factor the remaining quadratic function to find the remaining roots. The roots are 1, −1, −1, 5.

[1] correct answer without work shown

[0] incorrect answers and no work shown OR no answers given

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & -6 & 4 & 5 \\ & & -1 & 5 & 1 & -5 \\ \hline & 1 & -5 & -1 & 5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & -1 & 5 \\ & & 1 & -4 & -5 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

5-6 Enrichment

The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra tells us that a polynomial, $P(x)$, of degree $n \geq 1$ has exactly n complex roots. An equivalent statement is that a polynomial, $P(x)$, of degree $n \geq 1$ can be factored into n linear factors. Linear factors are the building blocks for every polynomial. Let's consider a similar theorem that applies to the building blocks of all positive integers. The Fundamental Theorem of Arithmetic states that every integer $N > 1$ can be uniquely written as a product of finitely many prime numbers.

1. Explain why $N > 1$ is a condition of the Fundamental Theorem of Arithmetic.
Answers may vary. Sample: The number 1 is not considered prime because it cannot be written as a product of 1 and another number.

2. Write the number 36 as a product of primes. **$36 = 2 \times 2 \times 3 \times 3$**

3. The Fundamental Theorem of Arithmetic tells us that there is only one way to write 36 as a product of prime numbers. The prime factors of 36 can be rearranged but will always be a form of $2^2 \times 3^2$. Write the number 48 as a product of primes. Express your answer using exponents. **$48 = 2^4 \times 3^1$**

Knowing the prime factorization of a number gives us complete knowledge of all of the factors of that number. For example, once we know that the prime factorization of 100 is $2^2 \times 5^2$ we know that other factors of 100 will be $2 \times 5^2 = 50$, $2^2 \times 5 = 20$, $2^0 \times 5 = 5$ or $2 \times 5^0 = 2$.

4. Write the number 600 as a product of primes expressed using exponents.
$600 = 2^3 \times 3^1 \times 5^2$

5. Use your prime factorization to determine all of the other factors of 600.
1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 25, 30, 40, 50, 60, 75, 100, 120, 150, 200, 300, 600

6. If the prime factorization can help you determine the factors of an integer, can the factored form of a polynomial help you determine nonlinear factors of a polynomial? For example, if $P(x) = (x - 1)(x + 2)(x - 5)$, then what are 3 quadratic factors of $P(x)$?
$(x - 1)(x + 2) = x^2 + x - 2$, $(x - 1)(x - 5) = x^2 - 6x + 5$, $(x + 2)(x - 5) = x^2 - 3x - 10$

7. Describe the similarities between the Fundamental Theorem of Algebra and the Fundamental Theorem of Arithmetic.
Answers may vary. Sample: Both theorems describe the number and types of factors. One addresses factors of a number and the other address factors of a polynomial.

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5-6 Reteaching

The Fundamental Theorem of Algebra

Problem

What are all the complex roots of $x^4 + x^3 - 2x^2 + 4x - 24 = 0$?

Because this is a fourth-degree polynomial, you know it will have four roots.

Step 1 Because the polynomial is already in standard form, you can use the Rational Root Theorem to determine possible rational roots. The possible rational roots are: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$.

Step 2 Evaluate the polynomial for each possible root until you find one that causes the polynomial to equal zero. This is a rational root. In this case, one rational root is 2.

Step 3 Use synthetic division with a divisor of 2 to begin factoring the polynomial.

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -2 & 4 & -24 \\ & & 2 & 6 & 8 & 24 \\ \hline & 1 & 3 & 4 & 12 & 0 \end{array}$$

$x^3 + 3x^2 + 4x + 12 = 0$

Step 4 Repeat Steps 1–3 until you have a polynomial of degree 2 or less.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 4 & 12 \\ & & -3 & 0 & -12 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4 = 0$

Step 5 If the dividend is a second-degree polynomial, factor to find any additional roots. If the dividend does not factor easily, use the Quadratic Formula to find the additional roots.

$$\frac{-0 \pm \sqrt{0^2 - 4(1)(4)}}{2(1)} = \frac{\pm\sqrt{-16}}{2} = \frac{\pm 4i}{2} = \pm 2i$$

The four roots of $x^4 + x^3 - 4x^2 + 2x - 24 = 0$ are 2, -3, $2i$, and $-2i$.

Exercises

Find all the complex roots of each polynomial.

1. $x^4 - 8x^3 + 11x^2 + 40x - 80$
   $4, \sqrt{5}, -\sqrt{5}$

2. $4x^4 - x^3 - 12x^2 + 4x - 16$
   $2, -2, \frac{1 + 3i\sqrt{7}}{8}, \frac{1 - 3i\sqrt{7}}{8}$

3. $x^6 + 2x^5 + 7x^4 + 20x^3 - 21x^2 + 18x - 27$
   $-3, 1, 3i, -3i, i, -i$

4. $x^3 - 4x^2 + 4x - 16$
   $4, 2i, -2i$

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5-6 Reteaching (continued)

The Fundamental Theorem of Algebra

Problem

What are the zeros of $f(x) = x^3 + 4x^2 - x - 10$?

The possible rational roots are $\pm1, \pm2, \pm5, \pm10$.

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -1 & -10 \\ & & 1 & 5 & 4 \\ \hline & 1 & 5 & 4 & -6 \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & 4 & -3 & -10 \\ & & -1 & -3 & 6 \\ \hline & 1 & 3 & -6 & -4 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -1 & -10 \\ & & 2 & 12 & 22 \\ \hline & 1 & 6 & 11 & 12 \end{array} \qquad \begin{array}{r|rrrr} -2 & 1 & 4 & -1 & -10 \\ & & -2 & -4 & 10 \\ \hline & 1 & 2 & -5 & 0 \end{array}$$

Use synthetic division to test each possible rational root until you get a remainder of zero.

So -2 is one of the roots.

$x^3 + 4x^2 - x - 10 = (x + 2)(x^2 + 2x - 5)$

Use the coefficients from synthetic division to obtain the quadratic factor.

Because $x^2 + 2x - 5$ cannot be factored, use the Quadratic Formula to solve $x^2 + 2x - 5 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{24}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{6}}{2}$$

$$x = -1 \pm \sqrt{6}$$

The polynomial function $f(x) = x^3 + 4x^2 - x - 10$ has one rational zero, -2, and two irrational zeros, $-1 + \sqrt{6}$ and $-1 - \sqrt{6}$.

Exercises

What are the zeros of each function?

5. $f(x) = x^3 - 2x^2 + 4x - 3$

$1, \frac{1 \pm i\sqrt{11}}{2}$

6. $f(x) = x^3 - 3x^2 - 15x + 125$

$-5, 4 \pm 3i$

7. $f(x) = 3x^3 - 2x^2 - 15x + 10$

$\frac{2}{3}, -\sqrt{5}, \sqrt{5}$

8. $f(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

$2, -2i, 2i$

9. $f(x) = x^4 - 3x^2 + 2$

$-1, 1, -\sqrt{2}, \sqrt{2}$

10. $f(x) = x^3 - 2x^2 - 17x - 6$

$-3, \frac{5 \pm \sqrt{33}}{2}$

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5-7 Additional Vocabulary Support

The Binomial Theorem

There are two sets of note cards that show how to expand the expression $(4x - 2)^4$. The set on the left explains the thinking. The set on the right shows the steps. Write the thinking and the steps in the correct order.

Think Cards

Rewrite the expression as a binomial sum.

Simplify the expansion.

Use the fourth row of Pascal's Triangle.

Apply the Binomial Theorem.

Write Cards

$(4x - 2)^4 = (4x + (-2))^4$

$(4x)^4 + 4(4x)^3(-2)^1 +$
$6(4x)^2(-2)^2 + 4(4x)^1(-2)^3 + (-2)^4$

$256x^4 - 512x^3 + 384x^2 - 128x + 16$

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ \boxed{1 \quad 4 \quad 6 \quad 4 \quad 1} \end{array}$$

Think	Write
First, rewrite the expression as a binomial sum.	**Step 1** $(4x - 2)^4 = (4x + (-2))^4$
Second, use the fourth row of Pascal's Triangle.	**Step 2** $\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ \boxed{1 \quad 4 \quad 6 \quad 4 \quad 1} \end{array}$
Next, apply the Binomial Theorem.	**Step 3** $(4x)^4 + 4(4x)^3(-2)^1 + 6(4x)^2(-2)^2 + 4(4x)^1(-2)^3 + (-2)^4$
Finally, simplify the expansion.	**Step 4** $256x^4 - 512x^3 + 384x^2 - 128x + 16$

5-7 Think About a Plan

The Binomial Theorem

Geometry The side length of a cube is given by the expression $(2x + 8)$. Write a binomial expression for the area of a face of the cube and the volume of the cube. Then use the Binomial Theorem to expand and rewrite the expressions in standard form.

Understanding the Problem

1. What is the formula for the area of a face of a cube? $A = s^2$

2. What is the formula for the volume of a cube? $V = s^3$

3. What is the problem asking you to determine?

expressions in standard form for the face area and volume of a cube with side length $(2x + 8)$

Planning the Solution

4. What is a binomial expression for the area of a face of this cube? $(2x + 8)^2$

5. What is a binomial expression for the volume of this cube? $(2x + 8)^3$

6. How can you use the Binomial Theorem to expand these expressions?

Answers may vary. Sample: Use $n = 2$ for the expression for area and use $n = 3$ for the expression for volume. For both expressions, $a = 2x$ and $b = 8$

____.

Getting an Answer

7. What is an expression for the area of a face of the cube written in standard form?

$4x^2 + 32x + 64$

8. What is an expression for the volume of the cube written in standard form?

$8x^3 + 96x^2 + 384x + 512$

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5-7 Practice

Form G

The Binomial Theorem

Expand each binomial. See answers below.

1. $(x + 2)^4$
2. $(a + 2)^7$
3. $(x + y)^7$
4. $(d - 2)^9$
5. $(2x - 3)^8$
6. $(x - 1)^9$
7. $(2x^2 - 2y^2)^6$
8. $(x^5 + 2y)^7$
9. $(n - 3)^3$
10. $(2n + 2)^4$
11. $(n - 6)^5$
12. $(n - 1)^6$
13. $(2a + 2)^3$
14. $(x^2 - y^2)^4$
15. $(2x + 3y)^5$
16. $(2x^2 + y^2)^6$
17. $(x^2 - y^2)^3$
18. $(2b + c)^4$
19. $(3m - 2n)^5$
20. $(x^3 - y^4)^6$

Find the specified term of each binomial expansion.

21. third term of $(x + 3)^{12}$ $594x^{10}$
22. second term of $(x + 3)^9$ $27x^8$
23. twelfth term of $(2 + x)^{11}$ x^{11}
24. third term of $(x - 2)^{12}$ $264x^{10}$
25. eighth term of $(x - 2y)^{15}$ $-823{,}680x^8y^7$
26. seventh term of $(x - 2y)^6$ $64y^6$
27. fifth term of $(x^2 + y^2)^{13}$ $715x^{18}y^8$
28. fourth term of $(x^2 - 2y)^{11}$ $-1320x^{16}y^3$
29. The term $126c^4d^5$ appears in the expansion of $(c + d)^n$. What is n? 9
30. The coefficient of the second term in the expansion of $(r + s)^n$ is 7. Find the value of n, and write the complete term. $n = 7$; $7r^6s$

State the number of terms in each expansion and give the first two terms.

31. $(d + e)^{12}$ 13; $d^{12} + 12d^{11}e$
32. $(x - y)^{15}$ 16; $x^{15} - 15x^{14}y$
33. $(2a + b)^5$ 6; $32a^5 + 80a^4b$
34. $(x - 3y)^7$ 8; $x^7 - 21x^6y$
35. $(4 - 2x)^8$ 9; $65{,}536 - 262{,}144x$
36. $(x^2 + y)^6$ 7; $x^{12} + 6x^{10}y$
37. The side of a number cube is $x + 4$ units long. Write a binomial for the volume of the number cube. Use the Binomial Theorem to expand and rewrite the expression in standard form. $(x + 4)^3 = x^3 + 12x^2 + 48x + 64$

1. $x^4 + 8x^3 + 24x^2 + 32x + 16$ 2. $a^7 + 14a^6 + 84a^5 + 280a^4 + 560a^3 + 672a^2 + 448a + 128$
3. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$
4. $d^9 - 18d^8 + 144d^7 - 672d^6 + 2016d^5 - 4032d^4 + 5376d^3 - 4608d^2 + 2304d - 512$
5. $256x^8 - 3072x^7 + 16{,}128x^6 - 48{,}384x^5 + 90{,}720x^4 - 108{,}864x^3 + 81{,}648x^2 - 34{,}992x + 6561$
6. $x^9 - 9x^8 + 36x^7 - 84x^6 + 126x^5 - 126x^4 + 84x^3 - 36x^2 + 9x - 1$
7. $64x^{12} - 384x^{10}y^2 + 960x^8y^4 - 1280x^6y^6 + 960x^4y^8 - 384x^2y^{10} + 64y^{12}$
8. $x^{35} + 14x^{30}y + 84x^{25}y^2 + 280x^{20}y^3 + 560x^{15}y^4 + 672x^{10}y^5 + 448x^5y^6 + 128y^7$
9. $n^3 - 9n^2 + 27n - 27$ 10. $16n^4 + 64n^3 + 96n^2 + 64n + 16$
11. $n^5 - 30n^4 + 360n^3 - 2160n^2 + 6480n - 7776$ 12. $n^6 - 6n^5 + 15n^4 - 20n^3 + 15n^2 - 6n + 1$
13. $8a^3 + 24a^2 + 24a + 8$ 14. $x^8 - 4x^6y^2 + 6x^4y^4 - 4x^2y^6 + y^8$
15. $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$
16. $64x^{12} + 192x^{10}y^2 + 240x^8y^4 + 160x^6y^6 + 60x^4y^8 + 12x^2y^{10} + y^{12}$ 17. $x^6 - 3x^4y^2 + 3x^2y^4 - y^6$
18. $16b^4 + 32b^3c + 24b^2c^2 + 8bc^3 + c^4$ 19. $243m^5 - 810m^4n + 1080m^3n^2 - 720m^2n^3 + 240mn^4 - 32n^5$
20. $x^{18} - 6x^{15}y^4 + 15x^{12}y^8 - 20x^9y^{12} + 15x^6y^{16} - 6x^3y^{20} + y^{24}$

5-7 Practice (continued) Form G

The Binomial Theorem

Expand each binomial.

38. $(x + 1)^7$
$x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$

39. $(x + 4)^8$
$x^8 + 32x^7 + 448x^6 + 3584x^5 + 17{,}920x^4 + 57{,}344x^3 + 114{,}688x^2 + 131{,}072x + 65{,}536$

40. $(x - 3y)^6$
$x^6 - 18x^5y + 135x^4y^2 - 540x^3y^3 + 1215x^2y^4 - 1458xy^5 + 729y^6$

41. $(x + 2)^5$
$x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

42. $(x^2 - y^2)^5$
$x^{10} - 5x^8y^2 + 10x^6y^4 - 10x^4y^6 + 5x^2y^8 - y^{10}$

43. $(3 + y)^5$
$y^5 + 15y^4 + 90y^3 + 270y^2 + 405y + 243$

44. $(x^2 + 3)^6$
$x^{12} + 18x^{10} + 135x^8 + 540x^6 + 1215x^4 + 1458x^2 + 729$

45. $(x - 5)^7$
$x^7 - 35x^6 + 525x^5 - 4375x^4 + 21{,}875x^3 - 65{,}625x^2 + 109{,}375x - 78{,}125$

46. $(x - 4y)^4$
$x^4 - 16x^3y + 96x^2y^2 - 256xy^3 + 256y^4$

47. Open-Ended Write a binomial in the form $(a + b)^n$ that has 3 as the coefficient of the first term. Answers may vary. Any binomial that has an a-term coefficient of $\sqrt[n]{3}$.

48. Use Pascal's Triangle to determine the binomial of the expanded expression $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$. $(x + 1)^6$

49. Error Analysis Your friend expands the binomial $(x - 2)^6$ as $x^6 + 12x^5 + 30x^4 + 160x^3 + 240x^2 + 192x + 64$. What mistake did your friend make? What is the correct expansion? Your friend used $b = 2$ instead of $b = -2$, and forgot to square the 2; $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

50. Reasoning Without writing any of the previous terms, how do you know that 2187 is the eighth term of the expansion of the binomial $(x + 3)^7$? According to Pascal's Triangle, the eighth term of a binomial with $n = 7$ is equal to $1a^0b^7$. In this binomial $a = x$ and $b = 3$. $1x^03^7 = 2187$.

51. In the expansion of $(3x - y)^6$, one of the terms contains the factor y^4.

a. What is the exponent of $3x$ in this term? 2

b. What is the coefficient of this term? 135

52. You are shipping a cubic glass sculpture. Each side of the sculpture is x in. long. To adequately protect the sculpture, the shipping box must leave room for 5 in. of padding on either side in every dimension. Write and expand a binomial for the volume of the shipping box. $V = (x + 5)^3 = x^3 + 15x^2 + 75x + 125$ in.^{3}

Name ______________________ Class ____________ Date ____________

5-7 Practice

Form K

The Binomial Theorem

Expand each binomial.

1. $(x + 4)^3$

To start, identify the third row of Pascal's Triangle. 1 3 3 1

$x^3 + 12x^2 + 48x + 64$

2. $(5 + a)^6$

$15{,}625 + 18750a + 9375a^2 + 2500a^3 + 375a^4 + 30a^5 + a^6$

3. $(y + 1)^4$

$y^4 + 4y^3 + 6y^2 + 4y + 1$

4. $(3a + 2)^4$

$81a^4 + 216a^3 + 216a^2 + 96a + 16$

5. $(x - 3)^5$

$x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

6. $(b + 1)^8$

$b^8 + 8b^7 + 28b^6 + 56b^5 + 70b^4 + 56b^3 + 28b^2 + 8b + 1$

7. $(x + 2)^3$

$x^3 + 6x^2 + 12x + 8$

Find the specified term of each binomial expansion.

8. second term of $(x - 4)^8$

$-32x^7$

9. third term of $(x + 3)^{12}$

$594x^{10}$

10. fourth term of $(x - 2)^7$

$-280x^4$

11. third term of $(x^2 - 2y)^6$

$60x^8y^2$

12. fifth term of $(3x - 1)^5$

$15x$

13. seventh term of $(x - 4y)^6$

$4096y^6$

14. third term of $(x^2 + y^2)^8$

$28x^{12}y^4$

15. second term of $(2 + x)^4$

$32x$

16. The term $56a^5b^3$ appears in the expansion of $(a + b)^n$. What is n?

8

17. The coefficient of the second term in the expansion of $(c + d)^n$ is 6. Find the value of n, and write the complete term.

$n = 6$; $6c^5d$

State the number of terms in each expansion and give the first two terms.

18. $(2a + b)^7$

8; $128a^7 + 448a^6b$

19. $(c - d)^8$

9; $c^8 - 8c^7d$

20. $(x + y)^3$

4; $x^3 + 3x^2y$

21. $(3x - y)^5$

6; $243x^5 - 405x^4y$

22. $(x + y^2)^5$

6; $x^5 + 5x^4y^2$

23. $(4 - 2x)^7$

8; $16{,}384 - 57{,}344x$

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5-7 Practice (continued) Form K

The Binomial Theorem

24. The side of a number cube is $x + 6$ units long. Write a binomial for the volume of the number cube. Use the Binomial Theorem to expand and rewrite the expression in standard form.
$(x + 6)^3 = x^3 + 18x^2 + 108x + 216$

Expand each binomial.

25. $(m + 1)^4$
$m^4 + 4m^3 + 6m^2 + 4m + 1$

26. $(2y + 8)^3$
$8y^3 + 96y^2 + 384y + 512$

27. $(2x + 2)^3$
$8x^3 + 24x^2 + 24x + 8$

28. $(x - 1)^8$
$x^8 - 8x^7 + 28x^6 - 56x^5$
$+ 70x^4 - 56x^3 + 28x^2 - 8x + 1$

29. $(x + 4)^5$
$x^5 + 20x^4 + 160x^3 + 640x^2$
$+ 1280x + 1024$

30. $(3b + 1)^6$
$729b^6 + 1458b^5 +$
$1215b^4 + 540b^3 + 135b^2 + 18b + 1$

31. Open-Ended Write a binomial in the form of $(a + b)^n$ that will have a first term coefficient equal to 7.
Any binomial such that a is the nth root of 7.

32. Use Pascal's Triangle to determine the binomial of the expanded expression $x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 56x^3 + 28x^2 + 8x + 1$.
$(x + 1)^8$

33. Error Analysis Your friend expands the binomial $(x - 4)^5$ as $x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024$. What mistake did your friend make? What is the correct expansion?
Your friend used $b = 4$ instead of $b = -4$; $x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x - 1024$

34. Reasoning Without writing any of the previous terms, how do you know that 64 is the seventh term of the expansion of the binomial $(x + 2)^6$.
According to Pascal's Triangle, the seventh term of a binomial with $n = 6$ is equal to $1a^0b^6$. In this binomial $a = x$ and $b = 2$. $1x^0 2^6 = 64$.

35. In the expansion of $(2m - n)^7$, one of the terms contains n^2.

a. What is the exponent of $2m$ in this term? 5

b. What is the coefficient of this term? 672

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5-7 Standardized Test Prep

The Binomial Theorem

Multiple Choice

For Exercises 1–7, choose the correct letter.

1. What is the expanded form of $(a - b)^3$? **D**

(A) $a^3 + a^2b + ab^2 + b^3$ (C) $a^3 + 3a^2b + 3ab^2 + b^3$

(B) $a^3 - a^2b + ab^2 - b^3$ (D) $a^3 - 3a^2b + 3ab^2 - b^3$

2. What is the third term in the expansion of $(x - y)^7$? **F**

(F) $21x^5y^2$ (G) $-7x^6y$ (H) $7x^6y$ (I) $-21x^5y^2$

3. What is the coefficient of the third term in the expansion of $(2x - y)^5$? **D**

(A) -80 (B) 32 (C) 40 (D) 80

4. Which term in the expansion of $(2a - 3b)^6$ has coefficient 2160? **G**

(F) second term (G) third term (H) fourth term (I) fifth term

5. What is n if $-448x^5y^3$ appears in the expansion of $(x - 2y)^n$? **C**

(A) 6 (B) 7 (C) 8 (D) 9

6. What is the 6th term in the 12th line of Pascal's Triangle? **H**

(F) 252 (G) 462 (H) 792 (I) 1287

7. What is the expanded form of $(2x - y)^5$? **B**

(A) $32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$

(B) $32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

(C) $2x^5 + 5x^4y + 20x^3y^2 + 20x^2y^3 + 10xy^4 + y^5$

(D) $2x^5 - 5x^4y + 20x^3y^2 - 20x^2y^3 + 10xy^4 - y^5$

Short Response

8. The coefficient of the fourth term in the expansion of $(x + y)^n$ is 84.

a. What is the value of n?

b. What is the complete term?

[2] $n = 9$; the complete term is $84(x)^6(y)^3 = 84x^6y^3$.

[1] incorrect number for n OR incorrect complete term

[0] no answers given

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5-7 Enrichment

The Binomial Theorem

Pascal's Triangle is helpful for expanding powers of a binomial sum, but what if you wanted to expand $(a + b)^{20}$? You may not want to write out all of the rows of Pascal's Triangle. In cases like this, you can use another form of the Binomial Theorem.

1. This form of the Binomial Theorem uses the operation called factorial. The expression 4! is read "four factorial" and is computed by multiplying all of the counting numbers from 4 down to 1. So, $4! = 4 \times 3 \times 2 \times 1 = 24$. What is 5!?
   $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

2. An unexpected fact arises for 0!. 0! is defined as 1. Fractions involving factorials can be simplified first. Rewrite each factorial in the fraction $\frac{6!}{3!}$ as multiplication and then reduce. What is the simplified form?

   $\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$; $6 \times 5 \times 4$

Expressions involving factorials can be used to write a different form of the Binomial Theorem. This form states that the k^{th} term in the expansion of $(a + b)^n$ is $\frac{n!}{(k - 1)!(n - k + 1)!}a^{n-k+1}b^{k-1}$.

3. Find the third term in the expansion of $(a + b)^8$ using the Binomial Theorem. What value will you substitute in for n? What value will you substitute in for k?

   $n = 8$; $k = 3$; $28a^6b^2$

4. Use Pascal's Triangle to verify that you have the found the correct third term.
   Answers may vary. Sample: The correct row is 1, 8, 28, 56, 70, 56, 28, 8, 1, and the third number, 28, is the correct coefficient.

Use the Binomial Theorem to find the indicated term of each expansion.

5. fifth term of $(x + y)^{15}$ **$1365x^{11}y^4$**

6. ninth term of $(x + 3)^{10}$ **$295{,}245x^2$**

7. twelfth term of $(2x + y)^{22}$ **$1{,}444{,}724{,}736x^{11}y^{11}$**

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5-7 Reteaching

The Binomial Theorem

You can find the coefficients of a binomial expansion in Pascal's Triangle.

To create Pascal's Triangle, start by writing a triangle of 1's. This triangle forms the first two rows. Each row has one more element than the one above it. Each row begins with a 1, and then each element is the sum of the two closest elements in the row above. The last element in each row is a 1.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Problem

What is the expansion of $(x + y)^5$? Use Pascal's Triangle.

Step 1 The power of the binomial corresponds to the second number in each row of Pascal's Triangle. Because the power of this binomial is 5, use the row of Pascal's Triangle with 5 as the second number. The numbers of this row are the coefficients of the expansion.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
$\boxed{1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1}$

Step 2 The exponents of the x-terms of the expansion begin with the power of the binomial and decrease to 0. The exponents of the y-terms of the expansion begin with 0 and increase to the power of the binomial.

Count down for *x*.

$1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$

Count up for *y*.

Step 3 Simplify all terms to write the expansion in standard form.

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Exercises

Write the expansion of each binomial.

1. $(a + b)^3$ $a^3 + 3a^2b + 3ab^2 + b^3$

2. $(x - y)^4$ $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

3. $(r + 1)^5$ $r^5 + 5r^4 + 10r^3 + 10r^2 + 5r + 1$

4. $(a - b)^6$ $a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$

5-7 Reteaching (continued)

The Binomial Theorem

- The *Binomial Theorem* states that for any binomial $(a + b)$ and any positive integer n, $(a + b)^n = {}_nC_0a^n + {}_nC_1a^{n-1}b + {}_nC_2a^{n-2}b^2 + \ldots + {}_nC_{n-1}ab^{n-1} + {}_nC_nb^n$.
- The theorem provides an effective method for expanding any power of a binomial.

Evaluate the combination ${}_nC_k$ as $\frac{n!}{k!(n-k)!}$.

Problem

What is the expansion of $(3x + 2)^3$? Use the Binomial Theorem.

Step 1 Determine a, b, and n.

$a = 3x, b = 2, n = 3$

Step 2 Use the formula to write the equation.

$(3x + 2)^3 = {}_3C_0(3x)^3 + {}_3C_1(3x)^2(2) + {}_3C_2(3x)(2)^2 + {}_3C_3(2)^3$

Step 3 Simplify.

$= 1(27x^3) + 3(9x^2)(2) + 3(3x)(4) - 1(8)$

$= 27x^3 + 54x^2 + 36x + 8$

Exercises

Fill in the correct coefficients, variables, and exponents for the expanded form of each binomial.

5. $(x + y)^4 = x^{\square} + \square x^3y + 6x^{\square}y^2 + \square xy^{\square} + \square^4$ **4; 4; 2; 4; 3; y**

6. $(z - y)^3 = z^{\square} - \square z^2y + \square zy^{\square} - \square^3$ **3; 3; 3; 2; y**

7. $(x + z)^5 = x^{\square} + \square x^4z + 10x^{\square}z^2 + \square x^2z^{\square} + \square xz^4 + \square^5$ **5; 5; 3; 10; 3; 5; z**

Write the expansion of each binomial. Use the Binomial Theorem.

8. $(x + y)^5$
$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

9. $(x - y)^5$
$x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

10. $(2x + y)^3$
$8x^3 + 12x^2y + 6xy^2 + y^3$

11. $(x + 3y)^4$
$x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$

12. $(x - 2y)^5$ $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

13. $(2x - y)^5$
$32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

14. $(x - 3y)^4$
$x^4 - 12x^3y + 54x^2y^2 - 108xy^3 + 81y^4$

15. $(4x - y)^3$
$64x^3 - 48x^2y + 12xy^2 - y^3$

16. $(x - 1)^5$
$x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$

17. $(1 - x)^3$
$1 - 3x + 3x^2 - x^3$

18. $(x^2 + 1)^3$ $x^6 + 3x^4 + 3x^2 + 1$

19. $(y^2 + a)^4$ $y^8 + 4y^6a + 6y^4a^2 + 4y^2a^3 + a^4$

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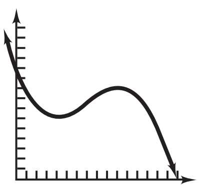
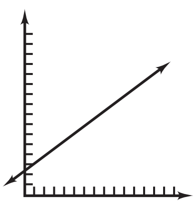
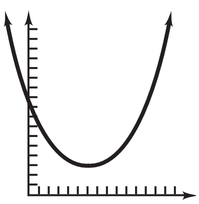
5-8 Additional Vocabulary Support

Polynomial Models in the Real World

Match each word phrase in Column A with the matching item in Column B.

Column A	Column B
1. interpolation	A. estimating outside the domain
2. $(n + 1)$ Point Principle	B. measures the fit between a model and a set of data
3. extrapolation	C. estimating within the domain
4. R^2 value	D. used to find a model that fits a set of data

Match each graph in Column A with the matching term in Column B.

Column A		Column B
5. 	C	A. linear
6. 	A	B. quadratic
7. 	B	C. cubic

8. Which form of estimation is more reliable: interpolation or extrapolation?

interpolation

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5-8 Think About a Plan

Polynomial Models in the Real World

Air Travel The table shows the percent of on-time flights for selected years. Find a polynomial function to model the data.

Year	1998	2000	2002	2004	2006
On-time Flights (%)	76.04	73.1	81.07	77.6	76.19

1. How can you plot the data? (*Hint*: Let x equal the number of years after 1990.)

subtract 1990 from each year to find the x-values; use the percent for the y-values

2. What types of models can you find for the data? **linear; quadratic; cubic; quartic**

3. Which model will fit the data points exactly? **quartic**

4. Find r^2 or R^2 for the models you listed in Exercise 2.

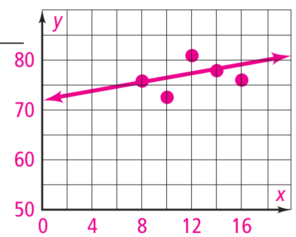
linear: 0.07; quadratic: 0.22; cubic: 0.45; quartic: 1.00

5. What does the value of r^2 or R^2 tell you about each model?

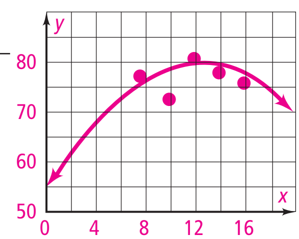
The quartic model is the best fit (an exact fit), followed by the cubic model, the quadratic model, and the linear model.

6. Graph each model on your graphing calculator. Sketch and label each model.

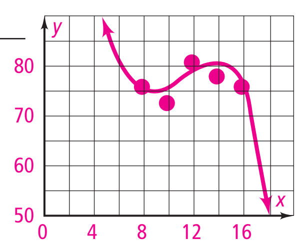
linear



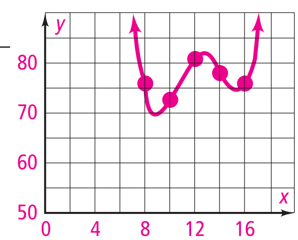
quadratic



cubic



quartic



7. Which model seems more likely to represent the percent of on-time flights over time?

Answers may vary. Sample: The linear model shows a gradual increase over time. Even though it has an r^2-value close to zero, it is most likely to represent the percent of on-time flights over time.

Name ______________________ Class ____________ Date ____________

5-8 Practice

Form G

Polynomial Models in the Real World

Find a polynomial function whose graph passes through each set of points.

1. (4, −1) and (−3, 13)
$y = -2x + 7$

2. $\left(1, -\frac{9}{2}\right)$ and (6, −2)
$y = \frac{1}{2}x - 5$

3. (7, −5) and (−1, 3)
$y = -x + 2$

4. (0, −3), (−2, −7), and (2, 9)
$y = x^2 + 4x - 3$

5. (−3, 15), (1, 11), and (0, 6)
$y = 2x^2 + 3x + 6$

6. (−2, −12), (1, −6), and (2, −24)
$y = -5x^2 - 3x + 2$

7. (4, −1), (−2, −13), and (1, 2)
$y = -x^2 + 4x - 1$

8. (0, 9), (2, 21) (−1, 0), and (3, 36)
$y = x^3 - 2x^2 + 6x + 9$

Find a polynomial function that best models each set of values.

9. Let x = the number of years after 1985.

World Gold

Year	Production (millions of troy ounces)
1985	49.3
1990	70.2
1995	71.8
2000	82.6

Sources: *The World Almanac* and *World Gold*

$f(x) = 0.038x^3 - 0.956x^2 + 8.01x + 49.3$

10. Let x = the number of years after 1970.

Life Expectancy

Year of Birth	Female (years)
1970	74.7
1980	77.4
1990	78.8
2000	79.7

Source: U.S. Bureau of the Census

$f(x) = 0.00013x^3 - 0.0105x^2 + 0.3617x + 74.7$

11. Let x = the number of years after 1985.

U.S. Energy

Year	Total Production ($\times 10^{15}$ Btu)
1985	64.9
1990	70.8
1995	71.0

Source: Energy Information Administration

$f(x) = -0.114x^2 + 1.75x + 64.9$

12. Let x = the number of years after 1980.

Social Security Benefits

Year	Monthly Average (dollars)
1980	321.10
1990	550.50
2000	844.60

Source: *www.infoplease.com*

$f(x) = 0.3235x^2 + 19.705x + 321.1$

Find a cubic and a quartic model for each set of values. Then determine which model best represents the values.

13.

x	−2	−1	0	1	2
y	−7	−3	3	5	−3

$f(x) = -x^3 - 2x^2 + 5x + 3;$
$f(x) = 0x^4 - x^3 - 2x^2 + 5x + 3;$
the cubic best represent the values.

14.

x	−2	−1	0	1	2
y	2	−6	2	8	42

$f(x) = x^3 + 5.86x^2 + 6x - 2.11;$
$f(x) = 2x^4 + x^3 - 3x^2 + 6x + 2;$
the quartic model best represents the values.

5-8 Practice (continued) — Form G

Polynomial Models in the Real World

Use your models from Exercises 9–12 to make predictions.

15. Estimate world gold production for 2010, 2020, and 2025.
245.8 troy oz., 787.8 troy oz., 1272.1 troy oz.

16. Estimate the life expectancy for women born in 1986, 1992, and 2005.
78.3 years, 79.0 years, 80.1 years

17. Estimate the U.S. energy production for 2002, 2005, and 2010.
61.7×10^{15} Btu, 54.3×10^{15} Btu, 37.4×10^{15} Btu

18. Estimate the average monthly Social Security benefits for 1970, 1996, and 1999.
\$156.40, \$719.20, \$812.28

19. Find a cubic function to model the data below. (*Hint*: Use x to represent the gestation period.) Then use the function to estimate the longevity of an animal with a gestation period of 151 days. **$0.0000006x^3 - 0.0005101x^2 + 0.1270416x + 2.0612682$; about 12 yr**

Gestation and Longevity of Certain Animals

Animal	Rat	Squirrel	Pig	Cow	Elephant
Gestation (in days)	21	44	115	280	624
Longevity (in years)	3	9	10	12	40

Source: *www.infoplease.com*

U.S. Population

Year	Foreign-Born (percent)
1910	14.7
1930	11.6
1950	6.9
1970	4.7
1990	8.0
2000	10.4
2004	11.7

Source: Bureau of the Census

20. Error Analysis Your teacher gives the class the table at the right and asks you to find a polynomial model for the data set. Then he asks the class to estimate the percent of U.S. foreign-born population in 1920. Your friend uses $x = -10$ and estimates the percent as 16.1. What did your friend do wrong? What is the correct estimate?
Your friend should have used $x = 10$ because 1920 is 10 years after the data set began not 10 years before. The correct estimate is 13.1%.

21. Reasoning Using the data set from Exercise 12 and the model you determined, find the average monthly Social Security benefits for the year 2050. Do you have much confidence in this prediction? Explain.
\$3285.60; no, because the data point is so far from the data set used to create the model. There is no way to predict what changes may occur between 2000 and 2050.

22. Find a cubic model for the following set of values: $(0, -4)$, $(-1, -6)$, $(5, -264)$, and $(2, -18)$. Using the regression coefficient, determine whether the model is a good fit.
$y = -2x^3 - x^2 + 3x - 4$; this is a good fit because $R^2 = 1$.

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5-8 Practice

Form K

Polynomial Models in the Real World

Find a polynomial function whose graph passes through each set of points.

1. $(-4, 31)$, $(2, 25)$, and $(0, 3)$

To start, substitute the x- and y-values of the points into the quadratic polynomial $y = ax^2 + bx + c$ to get three linear equations in three unknowns.

$$31 = a(-4)^2 + b(-4) + c$$
$$25 = a(2)^2 + b(2) + c$$
$$3 = a(0)^2 + b(0) + c$$

$3x^2 + 5x + 3$

2. $(1, 3)$ and $(4, -6)$

$y = -3x + 6$

3. $(-2, -19)$, $(0, 5)$, $(1, 8)$ and $(4, 53)$

$y = x^3 - 2x^2 + 4x + 5$

4. $(-4, -47)$, $(-1, 7)$, and $(1, 3)$

$y = -4x^2 - 2x + 9$

5. $(-3, 16)$, $(1, 0)$, and $(5, 16)$

$y = x^2 - 2x + 1$

6. $(-4, -3)$, $(-1, 12)$, $(0, 5)$ and $(1, 2)$

$y = x^3 + 2x^2 - 6x + 5$

7. $(-3, 8)$, $(0, -1)$, and $(2, 13)$

$y = 2x^2 + 3x - 1$

Find a polynomial function that best models each set of values.

8. Let $x = 0$.

Life Expectancy

Exact Age	Male (years)
5	70.5
10	65.6
15	60.6
20	55.9

SOURCE: 2004 U.S. Social Security

Answers may vary. Sample: $f(x) = -0.976x + 75.35$ or $f(x) = 0.002x^2 - 1.026x + 75.6$

9. Let x = the number of years since 1950.

World Silver

Year	Production (metric tons)
1950	6323
1955	9967
1960	7505
1965	8007

SOURCE: *The World Almanac & Book of Facts 2002*

$f(x) = 12.09x^3 - 303.52x^2 + 1944.07x + 6323$

5-8 Practice (continued) *Form K*

Polynomial Models in the Real World

Find a cubic and a quartic model for each set of values. Then determine which model best represents the values.

10.

x	−2	−1	0	1	2
y	−6	−2	2	7	−2

$f(x) = -1.17x^3 - 1.79x^2 + 5.67x + 3.37$;
$f(x) = -0.67x^4 - 1.17x^3 + 1.17x^2 + 5.67x + 2$;
the quartic model best represents the values.

11.

x	−2	−1	0	1	2
y	5	3	−5	−8	5

$f(x) = 1.83x^3 + 2.5x^2 - 7.33x - 5$;
$f(x) = 0x^4 + 1.83x^3 + 2.5x^2 - 7.33x - 5$;
the cubic model best represents the values.

12.

x	−2	−1	0	1	2
y	−10	−8	3	4	−3

$f(x) = -1.42x^3 - 2x^2 + 7.42x + 1.2$;
$f(x) = 0.88x^4 - 1.42x^3 - 5.88x^2 + 7.42x + 3$;
the quartic model best represents the values.

13.

x	−2	−1	0	1	2
y	4	−2	−1	0	2

$f(x) = -0.5x^3 + 1.14x^2 + 1.5x - 1.69$;
$f(x) = 0.33x^4 - 0.5x^3 - 0.33x^2 + 1.5x - 1$;
the quartic model best represents the values.

Use your models from Exercises 8 and 9 to make predictions.

14. Estimate the life expectancy for a 40 year-old male.

Answers may vary. Sample: using the linear model, the life expectancy is 36.31 yr. Using the quadratic model, the life expectancy is 37.76 yr.

15. Estimate world silver production for 2000.

Answers may vary. Sample: 855,977 metric tons

Name ______________________ Class ____________ Date ____________

5-8 Standardized Test Prep

Polynomial Models in the Real World

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which of the following is the polynomial function whose graph passes through (0, 4), (−2, 30), and (1, 6)? **B**

(A) $y = -9x + 10$
(C) $y = 9x - 10$
(B) $y = 5x^2 - 3x + 4$
(D) $y = -5x^2 + 3x - 4$

2. Which model type best represents the set of values at the right? **I**

x	−2	−1	0	1	2
y	−17	4	1	−2	−5

(F) linear
(H) quadratic
(G) cubic
(I) quartic

3. Which polynomial function best models the data set at the right? **D**

Paying Taxes for 1 Day

Year	Time Spent (minutes)
1940	83
1950	117
1960	130
1970	141
1980	145
1990	145
2000	160

Source: Tax Foundation

(A) $y = 0.00006006x^4 + 0.000119x^3 - 0.025x^2 + 2.13x + 71.6$
(B) $y = 0.00002163x^4 + 0.001267x^3 - 0.155x^2 + 8.24x + 81.2$
(C) $y = 0.00000312x^4 + 0.000197x^3 - 0.219x^2 + 5.22x + 86.3$
(D) $y = 0.00000606x^4 + 0.000217x^3 - 0.079x^2 + 3.90x + 83.5$

4. Using a cubic model for the data set at the right, what is the estimated Consumer Price Index for 1965? **H**

Consumer Prices

Year	Index
1920	60.0
1930	50.0
1940	42.0
1950	72.1
1960	88.7
1970	116.3
1980	248.8
1990	391.4
2000	515.8

Source: Bureau of Labor Statistics

(F) 102.5
(H) 116.564
(G) 130.034
(I) 147.384

Short Response

5. Find both a cubic and quartic model for the set of values at the right. Which model is a better fit? How do you know?

x	1.2	1.4	1.6	1.8	2.0	2.2
y	3.1	−4.2	4.1	7.5	−8.9	10

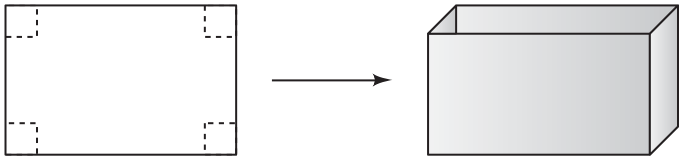
[2] cubic model $y = 62.27x^3 - 303.19x^2 + 481.81x - 248.52$ $R^2 = 0.142$; quartic model $y = 984.38x^4 - 6631.48x^3 + 16{,}498.68x^2 - 17{,}954.69x + 7208.88$ $R^2 = 0.936$; The quartic model is a better fit because R^2 is closer to 1.
[1] incorrect models OR incorrect analysis
[0] no answers given

Name ______________________ Class ____________ Date ____________

5-8 Enrichment

Polynomial Models in the Real World

You can represent volume with a cubic function. If you start with a rectangular piece of cardboard, cut out a square from each corner, and then fold up the sides, you will have an open-top rectangular solid.

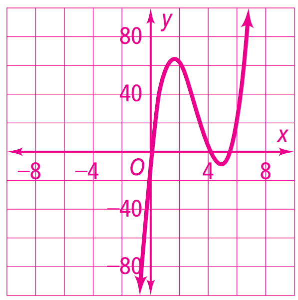


1. If you start with an 8.5 in.-by-11 in. piece of cardboard, what is the length, width, and height of the rectangular solid if you cut out a 1 in.-by-1 in. square from each corner?
length: 9 in.; width: 6.5 in.; height: 1 in.

2. What is the volume of the box?
58.5 in.^{3}

3. If you cut an x in.-by-x in. square from each corner, what is a general equation for the volume of the rectangular solid?
$V = x(11 - 2x)(8.5 - 2x)$

4. Sketch a graph of your volume equation.



5. Explain which part of your graph does not make sense for this real-world application.
Answers may vary. Sample: Any part of the graph that has negative values for *y* does not make sense, because the volume can never be less than zero.

6. What is the domain of this function? That is, what are the possible values for x, the size of square that can be cut out?
$0 < x < 4.25$

7. Explain why you cannot cut out squares that are larger than 4.25 in. on each side.
Answers may vary. Sample: The width of the cardboard is 8.5 in., so two squares measuring more than 4.25 in. would need more cardboard than is available.

8. What are the coordinates of the x-intercepts?
(0, 0), (4.25, 0), (5.5, 0)

9. What do the x-intercepts tell you about the rectangular box? **Answers may vary. Sample: The *x*-intercepts represent when the volume will be zero. This occurs when the box is flat, when no square has been cut out, or when the squares cut out equal the length or width (which is not possible).**

10. Use your graph to estimate the maximum possible volume.
approximately 66.1 in.^{3}

Name ____________________ Class __________ Date __________

5-8 Reteaching

Polynomial Models in the Real World

Problem

What polynomial function has a graph that passes through the four points $(0, -1)$, $(-1, -7)$, $(2, 17)$, and $(-2, -27)$?

Step 1 Use the $(n + 1)$ Point Principle to determine the degree (n) of the polynomial that fits these points perfectly. In this case $n + 1 = 4$, so the degree of the polynomial is 3 and the polynomial that fits these points will be $y = ax^3 + bx^2 + cx + d$.

Step 2 Substitute the x- and y-values from the four points given in the problem. Now you have four linear equations in four unknowns (a, b, c, d).

$a(0)^3 + b(0)^2 + c(0) + d = -1$ or $0a + 0b + 0c + d = -1$

$a(-1)^3 + b(-1)^2 + c(-1) + d = -7$ or $-a + b - c + d = -7$

$a(2)^3 + b(2)^2 + c(2) + d = 17$ or $8a + 4b + 2c + d = 17$

$a(-2)^3 + b(-2)^2 + c(-2) + d = -27$ or $-8a + 4b - 2c + d = -27$

Step 3 Enter the coefficients from the four linear equations as a matrix in your graphing calculator.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 1 & -7 \\ 8 & 4 & 2 & 1 & 17 \\ -8 & 4 & -2 & 1 & -27 \end{bmatrix}$$

Step 4 Recall that the **rref** function on your calculator returns the reduced row echelon form of a matrix. The last column of the reduced row echelon form is the solution to the system of equations represented by the matrix.

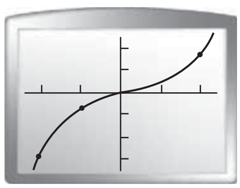
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Use the **rref** function to determine the coefficient values a, b, c, d.

$a = 2, b = -1, c = 3, d = -1$

So, the polynomial function is $y = 2x^3 - x^2 + 3x - 1$.

Step 5 Use the CubicReg calculator function with the four original points to check your answer.



Exercises

Find the polynomial function that passes through each set of points.

1. $(1, -4)$, $(-2, 38)$, $(0, 2)$, and $(-1, 10)$ $y = -3x^3 + x^2 - 4x + 2$

2. $(1, -5)$, $(-3, 39)$, and $(0, -6)$ $y = 4x^2 - 3x - 6$

Name ______________________ Class ____________ Date ____________

5-8 Reteaching (continued)

Polynomial Models in the Real World

Problem

The table shows winning times in the 400-meter run at a state track meet.

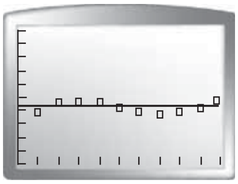
Track and Field 400-Meter Run										
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Seconds	53.86	54.66	54.72	54.60	54.27	53.87	53.70	53.89	54.14	54.62

Let $x =$ the number of years since 2001 and $y =$ the number of seconds.

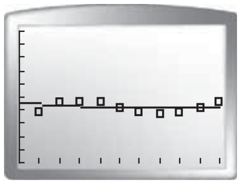
Determine whether a linear model, a quadratic model, or a cubic model best fits the data. Then use the model to estimate the winning time in 2018. Does your answer seem reasonable?

Find and graph a linear model, a quadratic model, and a cubic model for the data.

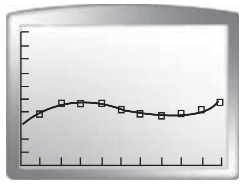
Linear model



Quadratic model



Cubic model



$y = -0.02455x + 54.36800$ $y = 0.0913x^2 - 0.12496x + 54.56883$ $y = 0.01961x^3 - 0.31443x^2 + 1.36733x + 52.88633$

Since the coefficient of the x^2-term in the quadratic model is almost 0, the graph of the quadratic model is very similar to the graph of the linear model.

The cubic model appears to be the best fit. Use it to estimate the winning time in 2013, or when $x = 12$. Substitute 12 for x and simplify.

$y = 0.01961(12)^3 - 0.31443(12)^2 + 1.36733(12) + 52.88633 \approx 57.9$

This estimate does seem to be reasonable because the times are in the mid-50's.

Exercises

3. The table shows winning points in men's springboard diving.

Men's Olympic Springboard Diving Records							
Year	1980	1984	1988	1992	1996	2000	2004
Points	905.02	754.41	730.80	676.53	701.46	708.72	787.38

Source: *www.infoplease.com*

a. Find a linear, quadratic, and cubic model for the data. Which model best fits the data?

b. Use the model of best fit to estimate the diving record in 2018. Does your estimate seem reasonable?

a. Let $x =$ the years since 1980 and $y =$ the points earned.
Linear: $y = -4.22893x + 802.79286$
Quadratic: $y = 1.08564x^2 - 30.28429x + 889.64405$
Cubic: $y = -0.0184939x^3 + 1.75142x^2 - 36.20234x + 896.74571$

b. The cubic model gives the best fit. In 2018, the estimated diving record is 1035.3 points. Answers may vary. Sample: The estimate does not seem reasonable. The most recent winning point values are in the 700s, but this estimate is much greater.

Name ______________________ Class ____________ Date ____________

5-9 Additional Vocabulary Support

Transforming Polynomial Functions

Choose a word from the list below to complete each sentence.

constant of proportionality	power function

1. A function written in the form $y = a \cdot x^b$ is called a power function.

2. The constant a in the function $y = ax^b$ is called the constant of proportionality.

Determine whether each of the following functions is a power function.

3. $y = 3x^2 - 4$ not a power function

4. $y = 4x^3$ power function

5. $y = 0.25x^6$ power function

6. $y = 7x^3 + 5x$ not a power function

7. $y = \frac{1}{3}x^5$ power function

Identify the constant of proportionality in each of the following functions.

8. $y = 5x^7$ 5

9. $y = 0.7x^3$ 0.7

10. $y = \frac{1}{5}x^2$ $\frac{1}{5}$

11. $y = 0.35x^8$ 0.35

Multiple Choice

12. Given the function $P = a \cdot w^2$, what is the value of a if $P = 225$ when $w = 5$? B

(A) 5 (B) 9 (C) 25 (D) 45

13. Given the function $P = a \cdot r^4$, what is the value of a if $P = 9$ when $r = 3$? F

(F) $\frac{1}{9}$ (G) $\frac{1}{5}$ (H) 9 (I) 81

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5-9 Think About a Plan

Transforming Polynomial Functions

Physics The formula $K = \frac{1}{2}mv^2$ represents the kinetic energy of an object. If the kinetic energy of a ball is 10 lb–ft^2/s^2 when it is thrown with a velocity of 4 ft/s, how much kinetic energy is generated if the ball is thrown with a velocity of 8 ft/s?

Know

1. The kinetic energy of a ball is $\boxed{\text{10 lb–ft}^2\text{/s}^2}$ when the velocity of the ball is $\boxed{\text{4 ft/s}}$.

2. **Check students' work.**

Need

3. To solve the problem I need to:

   solve the kinetic energy equation for *m*.

Plan

4. What equation can you use to find the value of m for the ball? $10 = \frac{1}{2}m(4)^2$

5. Solve the equation. $m = 1.25$

6. What equation can you use to find the kinetic energy generated if the ball is thrown with a velocity of 8 ft/s? $K = \frac{1}{2}(1.25)(8)^2$

7. Simplify. $K = 40$ lb–ft^2/s^2

8. Is the solution reasonable? Explain.

   Yes; the kinetic energy increases by a factor of 4 when the velocity doubles. The change in the kinetic energy is the square of the change in the velocity.

Name ______________________ Class ____________ Date ____________

5-9 Practice

Form G

Transforming Polynomial Functions

Determine the cubic function that is obtained from the parent function $y = x^3$ after each sequence of transformations.

1. a reflection in the x-axis;
a vertical translation 3 units down;
and a horizontal translation 2 units right
$y = -(x - 2)^3 - 3$

2. a vertical stretch by a factor of 4;
a reflection in the x-axis;
and a horizontal translation $\frac{1}{2}$ unit left
$y = -4\left(x + \frac{1}{2}\right)^3$

3. a vertical stretch by a factor of $\frac{1}{3}$;
a reflection in the y-axis;
and a vertical translation 6 units up
$y = \frac{1}{3}(-x)^3 + 6$

4. a vertical stretch by a factor of 3;
a reflection in the x-axis;
a vertical translation 2 units down;
and a horizontal translation 2 units left
$y = -3(x + 2)^3 - 2$

Find all the real zeros of each function.

5. $y = 2(x + 1)^3 - 3$ $x = \sqrt[3]{\frac{3}{2}} - 1$

6. $y = -3(x - 2)^3 + 24$ $x = 4$

7. $y = -\frac{1}{2}(x + 4)^3 - 1$ $x = \sqrt[3]{-2} - 4$

8. $y = 8(-x - 2)^3 + 5$ $x = \frac{\sqrt[3]{5}}{2} - 2$

9. $y = -(x + 5)^3 + 1$ $x = -4$

10. $y = 4(x - 6)^3 - 2$ $x = \frac{1}{\sqrt[3]{2}} + 6$

Find a quartic function with the given x-values as its only real zeros.

11. $x = 2$ and $x = 8$
Answers may vary. Sample:
$y = (x - 5)^4 - 81$

12. $x = 3$ and $x = -1$
Answers may vary. Sample:
$y = (x - 1)^4 - 16$

13. $x = 1$ and $x = 3$
Answers may vary. Sample:
$y = (x - 2)^4 - 1$

14. $x = -2$ and $x = 6$
Answers may vary. Sample:
$y = (x - 2)^4 - 256$

15. $x = 5$ and $x = -2$
Answers may vary. Sample:
$y = x^4 - 3x^3 - 9x^2 - 3x - 10$

16. $x = -1$ and $x = 2$
Answers may vary. Sample:
$y = x^4 - x^3 - x^2 - x - 2$

17. $x = -3$ and $x = -5$
Answers may vary. Sample:
$y = x^4 + 8x^3 + 16x^2 + 8x + 15$

18. $x = -4$ and $x = 4$
Answers may vary. Sample:
$y = x^4 - 15x^2 - 16$

19. Physics If you stretch a spring to 5 ft, it has 310 ft-lb of potential energy (PE). Potential energy varies directly as the square of the stretched length (l). The potential energy can be represented by the formula $PE = \frac{1}{2}kl^2$, where k is the spring constant.

a. What is the value of the spring constant for this spring? 24.8

b. How many ft-lbs of PE would an 8 ft length of spring have? 793.6 ft-lb

5-9 Practice (continued) Form G

Transforming Polynomial Functions

Determine whether each function can be obtained from the parent function $y = x^n$, using basic transformations. If so, describe the sequence of transformations.

20. $y = 2(x - 3)^3 + 4$ **yes; vertical stretch by a factor of 2, horizontal translation 3 units right, vertical translation 4 units up**

21. $y = x^4 + x - 3$ **no**

22. $y = -\frac{1}{3}x^2$ **yes; vertical stretch by a factor of $\frac{1}{3}$, reflection across x-axis**

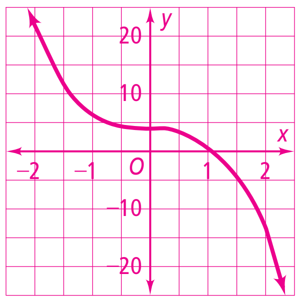
23. $y = (-x + 5)^3$ **yes; reflection across y-axis, horizontal translation 5 units right**

24. $y = \frac{2}{x^3}$ **yes; vertical stretch by a factor of 2**

25. $y = 4(x)^4 - 12$ **yes; vertical stretch by a factor of 4, vertical translation 12 units down**

26. Graph the parent function $y = x^3$ after it has been transformed by the following changes.

- vertical stretch by a factor of $2\frac{1}{4}$
- reflection across the x-axis
- vertical translation 4 units up



27. Error Analysis Your friend set up a problem to find a quartic function with the only real zeros of $x = -4$ and $x = 1$. She wrote down $y = (x + 4)(x - 1)(x^2 - 1)$. Will she get a correct quartic function? Why or why not?
No; she used a quadratic that has additional real zeros; she should have used $x^2 + 1$ as her quadratic.

28. Open-Ended Transform the parent function $y = x^3$ by vertical stretch, reflection across the x-axis, horizontal translation, and vertical translation.
Any cubic polynomial in the form of $y = a(x - h)^3 + k$ where $a < 0$ and h and $k \neq 0$.

29. You are swinging a bucket in a circle at a velocity of 7.8 ft/s. The radius of the circle you are making is 1.25 ft. The acceleration is equal to one over the radius multiplied by the velocity squared.

a. What is the acceleration of the bucket? **about 48.7 ft/s^2**

b. What is the velocity if the acceleration is 25 ft/sec^2? **about 5.6 ft/s**

Name ______________________________ Class ____________ Date ____________

5-9 Practice

Form K

Transforming Polynomial Functions

Determine the cubic function that is obtained from the parent function $y = x^3$ after each sequence of transformations.

1. a vertical stretch by a factor of 2
a vertical translation 5 units down;
and a horizontal translation 3 units left
To start, multiply by 2 to stretch. $y = 2x^3$
$y = 2(x + 3)^3 - 5$

2. a reflection across the x-axis;
a vertical translation 6 units up;
and a horizontal translation 4 units right
$y = -(x - 4)^3 + 6$

3. a vertical stretch by a factor of 3;
a reflection across the x-axis;
and a horizontal translation 6 units left
$y = -3(x + 6)^3$

4. a vertical stretch by a factor of $\frac{1}{2}$;
a reflection across the x-axis;
and a vertical translation 5 units down
and a horizontal translation 2 units left
$y = -\frac{1}{2}(x + 2)^3 - 5$

5. a vertical stretch by a factor of 2;
a reflection across the y-axis;
a vertical translation 2 units down
$y = 2(-x)^3 - 2$

Find all the real zeros of each function.

6. $y = 3(x - 1)^3 + 2$
$x = \sqrt[3]{\frac{-2}{3}} + 1$

7. $y = -5(x - 2)^3 + 20$
$x = \sqrt[3]{4} + 2$

8. $y = (x + 4)^3 - 1$
$x = -3$

9. $y = 5(-x + 1)^3 + 10$
$x = \sqrt[3]{2} + 1$

10. $y = 2(x - 5)^3 - 6$
$x = \sqrt[3]{3} + 5$

11. $y = -(x - 6)^3 + 1$
$x = 7$

Name ______________________ Class __________ Date __________

5-9 Practice (continued) *Form K*

Transforming Polynomial Functions

Find a quartic function with the given x-values as its only real zeros.

12. $x = 1$ and $x = 4$

To start, use the Factor Theorem to write the equation of a quartic with real roots at 1 and 4 and complex zeros where $Q(x)$ has zeros.

$$y = (x - 1)(x - 4) \cdot Q(x)$$

Answers may vary. Sample:
$y = x^4 - 5x^3 + 5x^2 - 5x + 4$

13. $x = -2$ and $x = -5$
Answers may vary. Sample:
$y = x^4 + 7x^3 + 11x^2 + 7x + 10$

14. $x = -3$ and $x = 2$
Answers may vary. Sample:
$y = x^4 + x^3 - 5x^2 + x - 6$

15. $x = 7$ and $x = 2$
Answers may vary. Sample:
$y = x^4 - 9x^3 + 15x^2 - 9x + 14$

16. $x = -3$ and $x = -4$
Answers may vary. Sample:
$y = x^4 + 7x^3 + 13x^2 + 7x + 12$

17. $x = -3$ and $x = -5$
Answers may vary. Sample:
$y = x^4 + 8x^3 + 16x^2 + 8x + 15$

18. $x = -5$ and $x = 5$
Answers may vary. Sample:
$y = x^4 - 24x^2 - 25$

19. You are swinging a bucket in a circle at a velocity of 8.2 ft/sec. The radius of the circle you are making is 1.5 ft. The acceleration is equal to one over the radius times the velocity squared.

a. What is the acceleration of the bucket? **44.8 ft/s^2**

b. What is the velocity if the acceleration is 28 ft/sec^2? **about 6.5 ft/s**

Name ______________________________ Class ______________ Date ______________

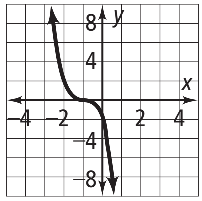
5-9 Standardized Test Prep

Transforming Polynomial Functions

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Which of the following describes the transformation of the parent function $y = x^3$ shown in the graph at the right? **A**



(A) reflection across x-axis, vertical stretch by a factor of 2, and horizontal translation 1 unit left

(B) reflection across y-axis, vertical translation 1 unit up

(C) horizontal translation 2 units left, vertical translation 1 unit down

(D) vertical stretch by a factor of $\frac{1}{2}$, horizontal translation 1 unit right, and vertical translation 2 units down

2. What are all the real zeros for $y = 5(x - 4)^3 + 6$? **H**

(F) $\sqrt[3]{\frac{-6}{5}} - 4$ (G) $\sqrt[3]{\frac{6}{5}} - 4$ (H) $\sqrt[3]{\frac{-6}{5}} + 4$ (I) $\sqrt[3]{\frac{6}{5}} + 4$

3. Which of the following polynomial functions *cannot* be obtained from the parent function $y = x^n$ using basic transformations? **D**

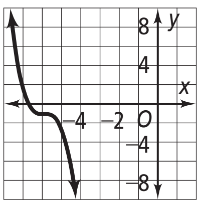
(A) $y = 6(x + 2)^3 - 3$ (B) $y = (-x - 1)^4$ (C) $y = \frac{x^2}{5}$ (D) $y = x^2 + x$

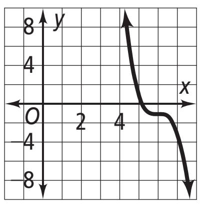
4. Which quartic function has $x = 3$ and $x = 9$ as its only real zeros? **G**

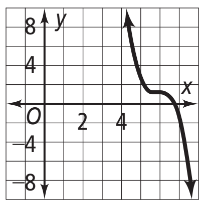
(F) $y = (x + 6)^4 - 81$ (H) $y = (x + 3)^4 - 81$

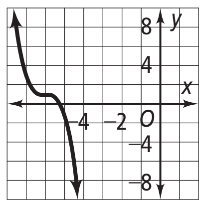
(G) $y = (x - 6)^4 - 81$ (I) $y = (x - 3)^4 - 81$

5. Which graph represents the polynomial $y = 2(-x + 6)^3 - 1$? **B**

(A)


(B)


(C)


(D)


Short Response

6. The formula $s = \frac{1}{2}at^2$ represents the distance an object will travel in a specific amount of time if it travels at a constant acceleration. You roll a ball 20 ft in 6 s. How long will it take to roll the ball 45 ft? Show your work.

[2] Substitute $s = 20$ and $t = 6$ into formula and solve for a; $20 = \frac{1}{2}a(6)^2$; $a \approx 1.11$ ft/s^2; substitute $s = 45$ and $a = 1.11$ into formula and solve for t; $t \approx 9$ s.

[1] incorrect time OR incorrect work shown

[0] no answers given

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5-9 Enrichment

Transforming Polynomial Functions

The discriminant of a quadratic equation is used to determine the number and type of solutions to a quadratic equation. A cubic equation also has a discriminant that determines the number and type of solutions to a cubic equation.

The general equation of a monic cubic equation that has a leading coefficient of 1 is

$y = x^3 + bx^2 + cx + d$

The discriminant of a monic cubic equation is $b^2c^2 - 4c^3 - 4b^3d - 27d^2 + 18bcd$. If

$b^2c^2 - 4c^3 - 4b^3d - 27d^2 + 18bcd > 0$, there are 3 real, distinct roots.

$b^2c^2 - 4c^3 - 4b^3d - 27d^2 + 18bcd = 0$, there are real roots and one has multiplicity 2.

$b^2c^2 - 4c^3 - 4b^3d - 27d^2 + 18bcd < 0$, there are 1 real and 2 complex, conjugate roots.

For each cubic, determine the value of the discriminant and state the number and type of roots.

1. $y = x^3 - 2x^2 + x + 7$ **−1351; 1 real and 2 complex, conjugate roots**

2. $y = x^3 + 4x^2 + 2x + 5$ **−1203;1 real and 2 complex, conjugate roots**

3. $y = x^3 - 7x^2 - x - 2$ **−3051; 1 real and 2 complex, conjugate roots**

4. $y = x^3 - 9x^2 - 1$ **−2943; 1 real and 2 complex, conjugate roots**

5. $y = x^3 + 10x^2 + 2x - 3$ **11,045; 3 real, distinct roots**

6. $y = x^3 + 5x^2 + 3x - 9$ **0; real roots with 1 of multiplicity 2**

7. $y = x^3 - 5x + 11$ **−2767; 1 real and 2 complex, conjugate roots**

8. $y = x^3 + 7x$ **−1372; 1 real and 2 complex, conjugate roots**

9. $y = x^3 + 5x^2$ **0; real roots with 1 of multiplicity 2**

10. $y = x^3 + 4x^2 - x - 4$ **900; 3 real, distinct roots**

5-9 Reteaching

Transforming Polynomial Functions

Problem

What is the equation of the graph of $y = x^3$ under the following transformations?

- vertical stretch by a factor of 4
- reflection across the x-axis
- horizontal translation 2 units left
- vertical translation 3 units up

Step 1 Begin by writing the general equation for stretching, reflecting, and/or translating the cubic parent function $y = a(x - h)^3 + k$.
a = vertical stretch h = horizontal translation k = vertical translation
If a function is reflected in the x-axis, a is negative.

Step 2 The vertical stretch is 4 and the transformed function is reflected across the x-axis.
$a = -4$

Step 3 The horizontal translation is 2 units left. This is the negative x direction, so h is negative.
$h = -2$

Step 4 The vertical translation is 3 units up. This is the positive y direction, so k is positive.
$k = 3$

Step 5 Substitute a, h, and k into the general equation.
$y = a(x - h)^3 + k = -4[x - (-2)]^3 + 3 = -4(x + 2)^3 + 3$

Exercises

Determine the equation of the graph of $y = x^3$ under each set of transformations.

1. a reflection across the x-axis, a vertical translation 5 units up, and a horizontal translation 8 units right **$y = -(x - 8)^3 + 5$**

2. a vertical stretch by a factor of $\frac{1}{4}$, a reflection across the y-axis, and a vertical translation 2 units down **$y = \frac{1}{4}(-x)^3 - 2$**

3. a vertical stretch by a factor of 6, a horizontal translation 3 units left, and a vertical translation 1 unit up **$y = 6(x + 3)^3 + 1$**

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5-9 Reteaching (continued)

Transforming Polynomial Functions

Problem

What is a quartic function with only two real zeros $x = -2$ and $x = 4$?

There are two methods you can use to find different quartic functions with only the two real zeros given.

The first method uses transformation.

Step 1 In this problem, you want zeros that are 6 units apart $(4 - (-2) = 6)$. Divide 6 in half because the basic quartic is centered on the y-axis.

Step 2 Raise this quotient to the fourth power (because you are trying to find a quartic function). This is how many units down you are translating the quartic. Because you are translating down, k will be negative. $3^4 = 81$, so $k = -81$.

Step 3 Once you vertically translate the parent quartic function down, the positive zero is located at 3 on the x-axis. The difference between 3 and 4 (the largest zero of the two given in the problem) is 1. You want to translate the quartic one unit to the right, so $h = 1$.

Step 4 Substitute the values for h and k into the general equation.
One quartic function with the desired zeros is $y = (x - 1)^4 - 81$.

You can also use an algebraic method to find a quartic function with the same zeros.

Step 1 Using the Factor Theorem, substitute the zeros into the factored form of the quartic function and multiply by $Q(x)$.
$y = (x + 2)(x - 4) \cdot Q(x)$

Step 2 $Q(x)$ is any quadratic that has no real zeros. To keep things simple, use $Q(x) = x^2 + 1$. Simplify the equation.
$y = (x + 2)(x - 4)(x^2 + 1)$
A different quartic function with the desired zeros is $y = x^4 - 2x^3 - 7x^2 - 2x - 8$.

Exercises

Find a quartic function with the given *x*-values as its only real zeros using transformations.

4. $x = -1$ and $x = 3$ $y = (x - 1)^4 - 16$

5. $x = 5$ and $x = 7$ $y = (x - 6)^4 - 1$

Find a quartic function with the given *x*-values as its only real zeros using the algebraic method.

6. $x = 4$ and $x = -2$
$y = x^4 - 2x^3 - 7x^2 - 2x - 8$

7. $x = -4$ and $x = 4$
$y = x^4 - 15x^2 - 16$

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Chapter 5 Quiz 1

Form G

Lessons 5-1 through 5-4

Do you know HOW?

Write each polynomial function in standard form. Then classify it by degree and by number of terms.

1. $n = 4m^2 - m + 7m^4$
$n = 7m^4 + 4m^2 - m$; quartic trinomial

2. $f(t) = 4t + 3t^3 + 2t - 7$
$f(t) = 3t^3 + 6t - 7$; cubic trinomial

3. $f(r) = 5r + 7 + 2r^2$
$f(r) = 2r^2 + 5r + 7$; quadratic trinomial

Find the zeros of each function. State the multiplicity of multiple zeros.

4. $y = (x + 2)^2(x - 5)^4$
-2, multiplicity 2; 5, multiplicity 4

5. $y = (3x + 2)^3(x - 5)^5$
$-\frac{2}{3}$, multiplicity 3; 5, multiplicity 5

6. $y = x^2(x + 4)^3(x - 1)$
0, multiplicity 2; -4, multiplicity 3; 1, multiplicity 1

Divide using synthetic division.

7. $(x^3 + 3x^2 - x - 3) \div (x - 1)$
$x^2 + 4x + 3$

8. $(2x^3 - 3x^2 - 18x - 8) \div (x - 4)$
$2x^2 + 5x + 2$

Find all the imaginary solutions of each equation by factoring.

9. $x^4 + 14x^2 - 32 = 0$
$\pm 4i$

10. $x^3 - 16x = 0$
none

11. $6x^3 - 2x^2 + 4x = 0$
$\frac{1 \pm i\sqrt{23}}{6}$

Do you UNDERSTAND?

12. What is $P(-4)$ given that $P(x) = 2x^4 - 3x^3 + 5x^2 - 1$?
$P(-4) = 783$

13. Open-Ended Write the equation of a polynomial function that has zeros at -3 and 2.
Answers may vary. Sample: $y = (x + 3)^2(x - 2)$

14. The product of three integers is 90. The second number is twice the first number. The third number is two more than the first number. What are the three numbers? 3, 6, 5

15. Reasoning The volume of a box is $x^3 + 4x^2 + 4x$. Explain how you know the box is not a cube.
The factors are $(x + 2)(x + 2)(x)$; these are the dimensions of the box; because they are not all the same, the box cannot be a cube.

16. Error Analysis For the polynomial function $y = \frac{1}{3}x^2 + x + 6$, your friend says the end behavior of the graph is down and up. What mistake did your friend make?
a is positive and n is even, so the end behavior of the graph is up and up.

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Chapter 5 Quiz 2

Form G

Lessons 5-5 through 5-9

Do you know HOW?

Expand each binomial.

1. $(2a - 1)^4$
$16a^4 - 32a^3 + 24a^2 - 8a + 1$

2. $(x + 3)^5$
$x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual rational roots.

3. $x^3 + 9x^2 + 19x - 4 = 0$
$\pm1, \pm2, \pm4; -4$

4. $2x^3 - x^2 + 10x - 5 = 0$
$\pm1, \pm5, \pm\frac{1}{2}, \pm\frac{5}{2}; \frac{1}{2}$

What are all the complex roots of the following polynomial equations?

5. $x^4 + 3x^3 - 5x^2 - 12x + 4 = 0$
$2, -2, \frac{-3 \pm \sqrt{13}}{2}$

6. $2x^3 + x^2 - 9x + 18 = 0$
$-3, \frac{5 \pm i\sqrt{23}}{4}$

7. Describe the transformations used to change the graph of the parent function $y = x^3$ to the graph of $y = \frac{1}{6}(x + 4)^3$.
vertical compression of $y = x^3$ by a factor of $\frac{1}{6}$, horizontal translation 4 units left

Find a polynomial function whose graph passes through each set of points.

8. $(0, 3), (-1, 0), (1, 10)$ and $(-2, -35)$ **$y = 6x^3 + 2x^2 - x + 3$**

9. $(-4, 215), (0, -1), (2, -1)$, and $(3, -16)$ **$y = -2x^3 + 5x^2 - 2x - 1$**

Do you UNDERSTAND?

10. The potential energy of a spring varies directly as the square of the stretched length l. The formula is $PE = \frac{1}{2}kl^2$, where k is the spring constant. When you stretch a spring to 12 ft, it has 483 ft-lb of potential energy. What is the spring constant for this spring? How much potential energy is created by stretching a 7 ft section?
$k = 6.7$; $PE = 164.2$ ft-lb

11. In the expansion of $(4r + s)^7$, one of the terms contains r^4s^3. What is the coefficient of this term? **8960**

12. Reasoning For a set of data, you make three models. R^2 for the quadratic model is 0.825. R^2 for the cubic model is 0.996. R^2 for the quartic model is 0.934. Explain why the cubic model may not be the best for predicting outside the data. **Answers may vary: Sample: You have to take into account the expected trend of the data to determine which model is the best.**

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Chapter 5 Test

Form G

Do you know HOW?

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. $4x^4 + 6x^3 - 2 - x^4$
$3x^4 + 6x^3 - 2$; quartic trinomial

2. $9x^2 - 2x + 3x^2$
$12x^2 - 2x$; quadratic binomial

3. $4x(x - 5)(x + 6)$
$4x^3 + 4x^2 - 120x$; cubic trinomial

Find the real solutions of each equation by graphing. Where necessary, round to the nearest hundredth.

4. $x^4 + 2x^2 - 1 = 0$
$-0.64, 0.64$

5. $-x^3 - 3x - 2 = 0$
-0.60

6. $-x^4 + 4x^3 + 3 = 0$
$-0.85, 4.05$

7. $-x^3 + 3x + 4 = 0$
2.20

8. $x^4 + 2x - 3 = 0$
$-1.57, 1$

9. $-x^3 + 2x^2 + 1 = 0$
2.21

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

10. 2, 3, 5
$y = x^3 - 10x^2 + 31x - 30$

11. $-1, -1, 1$
$y = x^3 + x^2 - x - 1$

12. $\sqrt{3}, 2i$
$y = x^4 + x^2 - 12$

13. $2-i, \sqrt{5}$
$y = x^4 - 4x^3 + 20x - 25$

Find the zeros of each function. State the multiplicity of any multiple zeros.

14. $y = (x - 1)^2(2x - 3)^3$
1, multiplicity 2; $\frac{3}{2}$, multiplicity 3

15. $y = (3x - 2)^5(x + 4)^2$
-4, multiplicity 2; $\frac{2}{3}$, multiplicity 5

16. $y = 4x^2(x + 2)^3(x + 1)$
-1, multiplicity 1; -2, multiplicity 3; 0, multiplicity 2

Solve each equation.

17. $(x - 1)(x^2 + 5x + 6) = 0$
$-3, -2, 1$

18. $x^3 - 10x^2 + 16x = 0$
$0, 2, 8$

19. $(x + 2)(x^2 + 3x - 40) = 0$
$-8, -2, 5$

20. $x^3 + 3x^2 - 54x = 0$
$-9, 0, 6$

Divide using synthetic division.

21. $(x^3 - 4x^2 + x - 5) \div (x + 2)$
$x^2 - 6x + 13$, $R - 31$

22. $(2x^3 - 4x + 3) \div (x - 1)$
$2x^2 + 2x - 2$, $R\ 1$

23. $(x^3 + 5x^2 - x + 1) \div (x + 2)$
$x^2 + 3x - 7$, $R\ 15$

24. $(3x^3 - x^2 + 2x - 5) \div (x - 1)$
$3x^2 + 2x + 4$, $R - 1$

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual roots.

25. $x^3 + 2x^2 + 3x + 6 = 0$
$\pm1, \pm2, \pm3, \pm6; -2$

26. $x^4 - 7x^2 + 12 = 0$
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12; -2, 2$

27. What is $P(-5)$ if $P(x) = -x^3 - 4x^2 + x - 2$? 18

Expand each binomial.

28. $(x + y)^4$
$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

29. $(4 - 3x)^3$
$64 - 144x + 108x^2 - 27x^3$

30. $(2r + q)^5$
$32r^5 + 80r^4q + 80r^3q^2 + 40r^2q^3 + 10rq^4 + q^5$

31. $(a + 4b)^3$
$a^3 + 12a^2b + 48ab^2 + 64b^3$

32.

Estimated Number of Deaths in the United States

Year	1960	1970	1980	1990	2000	2003
Deaths (millions)	1.71	1.92	1.99	2.15	2.40	2.42

SOURCE: *www.infoplease.com*

a. Find a cubic function to model the data. (Let x = years after 1960.) $y = 0.00001065x^3 - 0.000584x^2 + 0.02241x + 1.71758$

b. Estimate the deaths for the year 2006.
2.55 million

Determine the cubic function that is obtained from the parent function $y = x^3$ after each sequence of transformations.

33. a vertical stretch by a factor of 5, a reflection across the y-axis, and a horizontal translation 2 units left
$y = 5(-x - 2)^3$

34. a reflection across the x-axis, a horizontal translation 3 units right, and a vertical translation 7 units down
$y = -(x - 3)^3 - 7$

Do you UNDERSTAND?

35. Reasoning Would it be a good idea to use the cubic model found in Exercise 32 to estimate the deaths for the year 2050? Why or why not?
No; answers may vary. Sample: 2050 is far down the cubic model and you have no way of knowing what might affect the death rate in that many years.

36. Writing How do you use Pascal's Triangle when expanding a binomial? Pascal's Triangle gives you the coefficients of each term in the expanded binomial. You use the row in the triangle that has its second term the same as the binomial's exponent.

37. Can a function with the complex roots 5, $\sqrt{2}$, and $3i$ be a fourth-degree polynomial with rational coefficients? Explain. No; if a polynomial with complex roots 5, $\sqrt{2}$, and $3i$ has rational coefficients, then the complex factors of the polynomial must include $(x - 5)$, $(x - \sqrt{2})$, $(x + 3i)$, and $(x + 3i)$, so the polynomial must at least be a fifth-degree polynomial.

38. A cubic box is 5 in. on each side. If each dimension is increased by x in., what is the polynomial function modeling the new volume V?
$V = x^3 + 15x^2 + 75x + 125$ in.^{3}

Name ______________________________ Class ____________ Date ____________

Chapter 5 Quiz 1

Form K

Lessons 5–1 through 5–4

Do you know HOW?

Write each polynomial in standard form. Then classify it by degree and number of terms.

1. $-5t^2 + 9t + 8t^3$
$8t^3 - 5t^2 + 9t$; cubic trinomial

2. $5 + 3x^5 + 1 + x^5$
$4x^5 + 6$; quintic binomial

Find the zeros of each function. State the multiplicity of multiple zeros.

3. $y = (x - 3)^2(x + 4)^3$
3 multiplicity 2, −4 multiplicity 3

4. $y = x(x + 1)(x - 3)^4$
0 multiplicity 1, −1 multiplicity 1, 3 multiplicity 4

5. $y = (x + 1)^2(2x - 10)^4$
−1 multiplicity 2, 5 multiplicity 4

6. $y = (x - 4)^3(x + 3)$
4 multiplicity 3, −3 multiplicity 1

Find the real or imaginary solutions of each equation by factoring.

7. $x^3 - 2x^2 - 3x = 0$
0, −1, 3

8. $x^4 + 5x^2 = 6$
$\pm 1, \pm i\sqrt{6}$

Divide using synthetic division.

9. $(2x^3 - 6x^2 + 1) \div (x - 2)$
$2x^2 - 2x - 4$, R −7

10. $(x^3 + 5x^2 + 8x + 10) \div (x + 4)$
$x^2 + x + 4$, R −6

11. $(x^4 - 2x + 8) \div (x + 1)$
$x^3 - x^2 + x - 3$, R 11

12. $(3x^3 + 2x^2 - 4x + 1) \div (x - 2)$
$3x^2 + 8x + 12$, R 25

Do you UNDERSTAND?

13. Error Analysis For the polynomial function $y = -2x^3 + 2x - 5$, your friend says the end behavior of the graph is down and down. What mistake did your friend make? **a is negative and n is odd so the end behavior of the graph is up and down.**

14. Open-Ended Write a polynomial function in standard form with zeros −5, 3, and 2. **Answers may vary. Sample: $x^3 - 19x + 30$.**

Name ______________________ Class ____________ Date ____________

Chapter 5 Quiz 2

Form K

Lessons 5–5 through 5–9

Do you know HOW?

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual rational roots.

1. $x^3 - x^2 - 6x + 6 = 0$
±1, ±2, ±3, ±6; 1

2. $x^3 - x^2 + 5x - 7 = 0$
±1, ±7; none

3. $3x^3 + 2x^2 - x + 5 = 0$
$\pm 1, \pm 5, \pm\frac{1}{3}, \pm\frac{5}{3}$; none

4. $x^5 - 6x^4 + 4x^3 - 24x^2 + 3x - 18 = 0$
±1, ±2 ±3, ±6, ±9, ±18; 6

Without using a calculator, find all the roots of each equation.

5. $x^4 - 12x^2 + 27 = 0$
$3, -3, \sqrt{3}, -\sqrt{3}$

6. $x^5 + 3x^4 - 3x^3 - 9x^2 - 4x - 12 = 0$
$2, -2, -3, i, -i$

Expand each binomial

7. $(x + 2)^4$
$x^4 + 8x^3 + 24x^2 + 32x + 16$

8. $(3a - 1)^3$
$27a^3 - 27a^2 + 9a - 1$

Find a polynomial function whose graph passes through each set of points.

9. $(-4, 8)$, $(2, 14)$, and $(5, 44)$
$y = x^2 + 3x + 4$

10. $(-2, 10)$, $(-1, -4)$, $(0, -2)$, and $(1, -2)$
$y = -3x^3 - x^2 + 4x - 2$

Do you UNDERSTAND?

11. In the expansion of $(5x + y)^6$, one of the terms contains x^4y^2. What is the coefficient of this term?
9375

12. Compare and Contrast How are the graphs of $y = x^3$ and $y = (x + 3)^3$ alike? How are they different? What transformation was used to get the second equation? **Answers may vary. Sample: The graphs have the same shape and the same end behavior (down and up). The graph of $y = x^3$ has an x-intercept of 0, and the graph of $y = (x + 3)^3$ has an x-intercept of −3; translation 3 units left**

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Chapter 5 Test

Form K

Do you know HOW?

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. $6x^5 - 2x^2 + 1 - 2x^5$
$4x^5 - 2x^2 + 1$; quintic trinomial

2. $x^2 - 3x + 6x^3 - 5x + 1$
$6x^3 + x^2 - 8x + 1$; cubic, 4 terms

Determine the end behavior of the graph of each polynomial function.

3. $y = 3x + 2x^2 - 4$
up and up

4. $y = 4x^3 - 7x + 2$
down and up

Find the zeros of each function. State the multiplicity of multiple zeros.

5. $y = (x + 2)(x - 3)^2$
-2 multiplicity 1; 3 multiplicity 2

6. $y = x^3 + 4x^2$
0 multiplicity 2; -4 multiplicity 1

Find the real solutions of each equation using a graphing calculator. Where necessary, round to the nearest hundredth.

7. $5x^3 - 2x^2 - 1 = 0$
0.75

8. $2x^4 + 4x^2 = 4$
-0.86, 0.86

Divide using long division. Check your answers.

9. $(x^2 + 6x + 24) \div (x + 4)$
$x + 2$, R 16

10. $(x^3 + 2x^2 + 4x + 10) \div (x + 1)$
$x^2 + x + 3$, R 7

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

11. $-1, 2, 6$
$P(x) = x^3 - 7x^2 + 4x + 12$

12. $-i, \sqrt{2}$
$P(x) = x^4 - x^2 - 2$

Find all the zeros of each function.

13. $y = x^3 + 2x^2 + 4x + 8$
$-2, -2i, 2i$

14. $y = x^4 - 14x^2 + 45$
$-3, 3, -\sqrt{5}, \sqrt{5}$

Expand each binomial.

15. $(x + 2)^6$

$x^6 + 12x^5 + 60x^4 + 160x^3$
$+ 240x^2 + 192x + 64$

16. $(2x + 3)^5$

$32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$

17. Find a cubic function to model the data in the table. Let x represent years after 1990.

$y \approx -0.00073332x^3 + 0.0202x^2 - 0.1407x + 4.18$

Births in the United States

Year	Births (millions)
1990	4.16
1995	3.89
2000	4.06
2005	4.14

SOURCE: www.cdc.gov

Determine the cubic function that is obtained from the parent function $y = x^3$ after each sequence of transformations.

18. a vertical stretch by a factor of 3;
a reflection across the y-axis;
and a horizontal translation 4 units right

$y = -3(x - 4)^3$

19. a reflection across the x-axis; a horizontal translation 2 units left; and a vertical translation 6 units down

$y = -(x + 2)^3 - 6$

Do you UNDERSTAND?

20. The product of three integers is 56. The second number is twice the first number. The third number is five more than the first number. What are the three numbers?

2, 4, 7

21. What is $P(2)$ given that $P(x) = 3x^4 - x^3 + 2x^2 - 10$? Use synthetic division and the Remainder Theorem.

$P(2) = 38$

22. Open-Ended Write a polynomial function of degree 3 with rational coefficients and exactly one real zero. List all of the zeros of the function.

Answers will vary. Sample: $y = x^3 - 3x^2 + 5x - 15$; zeros: $3, i\sqrt{5}, -i\sqrt{5}$

21. A cubic box is 4 in. on each side. If each dimension is increased by $2x$ in., what is the polynomial function modeling the new volume V?

$V = 8x^3 + 48x^2 + 96x + 64$ in.^{3}

Name ______________________________ Class ____________ Date ____________

Chapter 5 Performance Tasks

Task 1

a. Draw the related graph of $x^2 - ax = bx - ab$. Determine the multiplicity of each root.

b. Draw the related graph of $(x - a)^2(x - b) = 0$. Determine the multiplicity of each root.

c. Rewrite the equations found in parts a and b in standard form.

d. Given the equation $ax^3 + bx^2 = -cx$, find the roots of this equation in terms of a, b, and c.

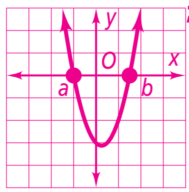
[4] All parts of task completed correctly with graphs and work shown OR used correct process with minor computational errors.

[3] Student graphed equations incorrectly but followed through the process correctly using the wrong data.

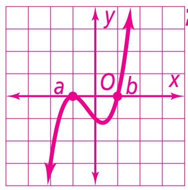
[2] Correct answer, without work shown OR student only able to complete part of task.

[1] Student understood that graphing was involved but was unable to correctly graph the equations or solve for equation roots.

[0] No attempt was made to solve this problem OR answer is incorrect, without work shown.

a. 

; *a*, multiplicity 1, *b*, multiplicity 1

b. 

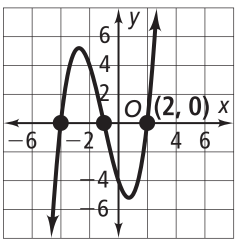
; *a*, multiplicity 2, *b*, multiplicity 1

c. $x^2 - (a + b)x + ab = 0$; $x^3 - (2a + b)x^2 + (a^2 + 2ab)x - a^2b = 0$

d. $0, \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Task 2

a. Use division to find the remaining roots of $y = \frac{1}{2}x^3 + \frac{3}{2}x^2 - 3x - 4$.



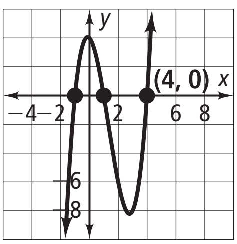
a. −1, −4 b. 1, −1

c. $y = \frac{1}{2}(x - 2)(x + 1)(x + 4)$ **and** $y = (x - 4)(x - 1)(x + 1)$

[4] All parts of task completed correctly with equations and work shown OR used correct process with minor computational errors.

[3] Student divided equations incorrectly but followed through the process correctly using the wrong data.

b. Use division to find the remaining roots of $y = x^3 - 4x^2 - x + 4$.



[2] Correct answer, without work shown OR student only able to complete part of task.

[1] Student understood that division was involved but was unable to correctly determine the dividend and divisor or solve for quotient.

[0] No attempt was made to solve this problem OR answer is incorrect, without work shown.

c. Use the roots found in parts a and b to rewrite the functions in factored form.

Chapter 5 Performance Tasks (continued)

Task 3

The data in the table at the right shows the times for the Men's 500-m Speed Skating event at the Winter Olympics.

Year	Time (sec)
1984	38.19
1988	36.45
1992	37.14
1994	36.33
1998	35.59
2002	34.42
2006	34.84

SOURCE: *www.infoplease.com*

a. Find a quadratic, a cubic, and a quartic model for the data set. Let x be the number of years since 1980.

b. Compare the models and determine which one is more appropriate. Explain your choice.

[4] quadratic model $y = 0.00122x^2 - 0.17887x + 38.60857$, $R^2 = 0.855$; cubic model $y = 0.0000998x^3 - 0.00357x^2 - 0.11338x + 38.37857$, $R^2 = 0.855$; quartic model $y = 0.0002x^4 - 0.01288x^3 + 0.27695x^2 - 2.44290x + 44.25429$ $R^2 = 0.974$; Even though the quartic model is technically the best fit because its R^2 is the closest to 1, it is unlikely that skating times would drop as quickly as the quartic model shows. The quadratic model shows a model that represents the real-world data better with a more gradual lowering of skating times. Student shows a full understanding of the processes, even if there are minor computational errors.

[3] Student miscalculated model regressions, but understood how R^2 being close to 1 does not always lead to the more appropriate model. The student must also look at the data for realistic changes.

[2] Correct answer, without work shown OR student able to complete part of the task.

[1] Student understood the need to use the data to obtain model regressions, but unable to accomplish this.

[0] No attempt was made to solve this problem OR answer is incorrect, without work shown.

Task 4

The power P generated by a circuit varies directly to the square of the current I times the resistance R.

a. Write quadratic functions that model circuits with a power of 15 watts at 6 amps current, of 30 watts at 12 amps current, and of 60 watts at 24 amps current.

b. Find the zeros of the functions.

c. What does each zero represent?

[4] $P = I^2R$; $0 = 36R_1 - 15$; $0 = 144R_2 - 30$, $0 = 576R_3 - 60$; $R_1 = 0.417$; $R_2 = 0.208$; $R_3 = 0.104$; These zeros are the resistance for each of the three circuits. Student shows a full understanding of the processes, even if there are minor computational errors.]

[3] Student incorrectly set up functions, but understood how to find the zeros.

[2] Correct answer, without work shown OR student able to complete part of the task.

[1] Student understood the need to use functions, but unable to accomplish this.

[0] No attempt was made to solve this problem OR answer is incorrect, without work shown.

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Chapter 5 Cumulative Review

Multiple Choice

For Exercises 1–14, choose the correct letter.

1. Which relation is *not* a function? D

   (A) $y = 0$ (B) $y = 2x$ (C) $y = x + 2$ (D) $x = 2$

2. For which of the following sets of data is a linear model reasonable? F

   (F) $\{(0, 11), (2, 8), (3, 7), (7, 2), (8, 0)\}$

   (G) $\{(-15, 8), (-8, -7), (-3, 0), (0, 5), (7, -3)\}$

   (H) $\{(-10, 3.5), (-5.5, 6.5), (-0.1, -4), (3.5, -7.5), (12, -5)\}$

   (I) $\{(-1, 3.5), (0, 2.5), (2, 6.5), (-3, 11.5), (5, 27.5)\}$

3. Which is a solution of the system of inequalities $\begin{cases} y + 4 > 0 \\ y \leq x + 1 \end{cases}$? A

   (A) $(3, 3)$ (B) $(-1, 2)$ (C) $(1, 5)$ (D) $(0, 2)$

4. Which of the following is the equation of a parabola? H

   (F) $y = x - 1$ (G) $y = |x + 3|$ (H) $y = x^2 + 1$ (I) $x = y + 2$

5. Which of these is a direct variation? C

   (A) $x = 8$ (B) $y = 8$ (C) $y = 8x$ (D) $y = 8x^2$

6. Which of these quadratic equations has the factors $(x - 2)$ and $(x - 3)$? H

   (F) $x^2 - x - 6$ (G) $x^2 + x - 6$ (H) $x^2 - 5x + 6$ (I) $x^2 + 5x + 6$

7. Which polynomial is written in standard form? D

   (A) $1 + 3x - 5x^2$ (B) $3x^2 + 2 + x^3$ (C) $4x - 5x^2$ (D) $6x^3 - x + 7$

8. Solve the system $\begin{cases} x + 4 = 0 \\ y = x + 1 \end{cases}$. F

   (F) $(-4, -3)$ (G) $(4, 3)$ (H) $(-4, -5)$ (I) $(-4, 3)$

9. Solve $8x < 12 + 4x$. B

   (A) $x < 1$ (B) $x < 3$ (C) $x > 3$ (D) $x = 3$

10. What is the axis of symmetry of $y = 2(x - 3)^2 + 5$? H

    (F) $y = 3$ (G) $x = 2.5$ (H) $x = 3$ (I) $x = 5$

Chapter 5 Cumulative Review (continued)

11. A sixth-degree polynomial function with rational coefficients has complex roots 6, $\sqrt{2}$, and $-5i$. Which of the following *cannot* be another complex root of this polynomial? **B**

(A) $5i$ (B) $i\sqrt{3}$ (C) $-\sqrt{2}$ (D) 0

12. Solve $(x + 3)(x + 4) = 0$ **H**

(F) $x = 3$ or $x = 4$ (H) $x = -3$ or $x = -4$

(G) $x = 0$ (I) none of the above

13. Which relation is a function? **B**

(A) $\{(2, 3), (3, 5), (1, 4), (2, -1)\}$ (C) $\{(3, 1), (3, 3), (3, 2), (3, 0)\}$

(B) $\{(1, 0), (0, 2), (3, 9), (-1, 8)\}$ (D) $\{(1, 4), (2, 4), (4, 3), (4, 4)\}$

14. Find the roots of $x^3 + x^2 - 17x + 15 = 0$. **I**

(F) 1, 3, 5 (G) $-5, -3, -1$ (H) $-1, 3, 5$ (I) $-5, 1, 3$

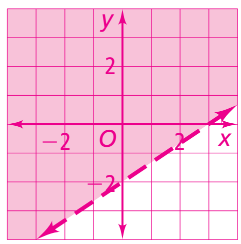
Short Response

15. Open-Ended Write the equation of a direct variation in slope-intercept form. Write the x-and y-intercepts. **Answers may vary. Sample: $y = 3x$, (0, 0)**

16. Writing Explain how to write a polynomial equation in standard form with roots $x = a, b, c$. **Check student's work.**

17. Evaluate $2a^2 - 5a + 4$ for $a = 3$. **7**

18. Graph the inequality: $2x - 3y < 6$.



19. Use Pascal's Triangle or the Binomial Theorem to expand $(x - y^2)^3$.
$x^3 - 3x^2y^2 + 3xy^4 - y^6$

20. Determine the equation of the graph of $y = x^3$ under a vertical stretch by a factor of 8, a reflection across the x-axis, a horizontal translation 3 units left, and a vertical translation 5 units up. **$y = -8(x + 3)^3 + 5$**

Extended Response

[4] a. h will equal zero when the arrow hits the ground, so $0 = -16t^2 + 32t + 5$; $t = 2.15$ s; b. $h = -16(2)^2 + 32(2) + 5$, $h = 5$ ft
[3] correct equations, but with one computational error

21. An arrow is shot upward. Its height h, in feet, is given by the equation $h = -16t^2 + 32t + 5$, where t is the time in seconds. The arrow is released at $t = 0$ s.

a. How many seconds does it take until the arrow hits the ground?

b. How high is the arrow after 2 seconds? **[2] incorrect equations OR multiple computational errors**
[1] correct answers, without work shown
[0] incorrect answers and no work shown OR no answers given

Chapter 5 Project Teacher Notes: Curves by Design

About the Project

The Chapter Project gives students an opportunity to adjust a polynomial equation to fit the curve for their designs of the hood section of a car. They also write cubic equations for curves of objects of their choice by using inverse matrices and by using their calculator's regression feature.

Introducing the Project

- Encourage students to keep all project-related materials in a separate folder.
- Ask students if they have ever wondered how car designers change the shapes of a car's parts. Ask students what they think an equation for a curved section of a car would look like.

Activity 1: Graphing

Students graph the given polynomial and fine-tune the equation to make the graph a pleasing shape for a car hood.

Activity 2: Analyzing

Students research the designs of cars or other objects that have curved parts and use inverse matrices to write equations for one of their curves.

Activity 3: Graphing

Students use their calculators to find more accurate equations to model the curves for their projects.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results. Ask students to review their project work and update their folders.

- Have students review their methods for finding and recording curves and equations used for the project.
- Ask groups to share their insights that resulted from completing the project, such as techniques they found to make fitting the equations to the curves easier or more accurate.

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Chapter 5 Project: Curves by Design

Beginning the Chapter Project

A continuous curve can be approximated by the graph of a polynomial. This fact is central to modern car design. Scale models are first produced by a designer. Even such apparently minor parts of the design such as door handles are included in models.

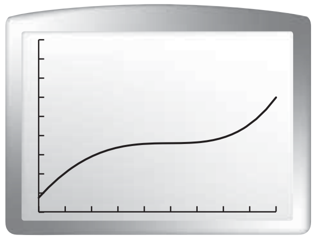
When the modeling process is complete, every curve in the design becomes an equation that is adjusted by the designer on a computer. Minor changes can be made through slight changes in an equation. Although in many programs the computer adjusts the equations, you can do the same thing on a graphing calculator. When the design has been finalized, the information is used to produce dies and molds to manufacture the car.

List of Materials

- Graphing calculator
- Graph paper

Activities

Activity 1: Graphing



A hood section of a new car is modeled by the equation $y = 0.00143x^4 + 0.00166x^3 - 0.236x^2 + 1.53x + 0.739$. The graph of this polynomial equation is shown at the right. Use a graphing calculator to fine-tune the equation. Keep the same window but change the equation. Pretend you are the designer and produce a curve with a shape more pleasing to your eye! **Check students' work.**

Activity 2: Analyzing

Research the design of a car or another object that has curved parts.

- On graph paper, sketch a curve that models all or part of the object you chose to research. Label four points that you think would help identify the curve. Find the cubic function that fits these four points.
- Use the equation $y = ax^3 + bx^2 + cx + d$. Solve for the variables a, b, c, and d using a 4×4 inverse matrix.

Check students' work.

Activity 3: Graphing

Identify and label ten points on the sketch you made in Activity 2. Do you think the function that best fits these points will be more accurate than the function you found using four points? Explain your reasoning. Then find the new function using a graphing calculator and the CubicReg feature. **Check students' work.**

Chapter 5 Project: Curves by Design (continued)

Finishing the Project

The activities should help you to complete your project. Make a poster to display the sketch and graphs you have completed for the object you have chosen. On the poster, include your research about the object.

Reflect and Revise

Before completing your poster, check your equations for accuracy, your graph designs for neatness, and your written work for clarity. Is your poster eye-catching, exciting, and appealing, as well as accurate? Show your work to at least one adult and one classmate. Discuss improvements you could make.

Extending the Project

Interview someone who uses a computer-assisted design (CAD) program at work. If possible, arrange to have a demonstration of the program. Find out what skills, education, or experience helped the person successfully enter the field of computer-assisted design.

Chapter 5 Project Manager: Curves by Design

Getting Started

Read the project. As you work on the project, you will need a calculator, materials on which you can record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist

- ☐ Activity 1: modeling a curve
- ☐ Activity 2: finding a cubic model
- ☐ Activity 3: finding a better fit
- ☐ object model

Suggestions

- ☐ Make small changes in the equation at first.
- ☐ Label the turning points.
- ☐ Use the regression feature of your graphing calculator.
- ☐ Is a cubic function the best model for the object you chose? Why or why not? How can you determine the curve that best models the shape of your object using a graphing calculator?

Scoring Rubric

4 Your equations and solutions are correct. Graphs are neat and accurate. All written work, including the poster, is neat, correct, and pleasing to the eye. Explanations show careful reasoning.

3 Your equations are fairly close to the graph designs, with some minor errors. Graphs, written work, and the poster are neat and mostly accurate with minor errors. Most explanations are clear.

2 Your equations and solutions contain errors. Graphs, written work, and the poster could be more accurate and neater. Explanations are not clear.

1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.

0 Major elements of the project are incomplete or missing.

Your Evaluation of Project Evaluate your work, based on the *Scoring Rubric.*

Teacher's Evaluation of Project